DARBHANGA COLLEGE OF ENGINEERING

DARBHANGA

ENGINEERING MECHANICS

DEPARTMENT OF CIVIL ENGINEERING

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DCE, Darbhanga

Engineering Mechanics

ESC205

L	T	P	Cr
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Course Objectives:

1. Understand the scalar representation of forces and moments.
2. Describe static equilibrium of particles and rigid bodies in two dimensions including the effect of friction.
3. Analyse the properties of surfaces and solids in relation to moment of inertia.
4. Illustrate the laws of motion, kinematics of motion and their relationship.
5. Study the mechanical vibration without and with damping of SODF and MDOF.

Course Outcomes:

After successful completion of this course, the students should be able to:

CO1: Construct free body diagram and calculate the reactions necessary to ensure static equilibrium.

CO2: Study the effect of friction in static and dynamic conditions.

CO3: Understand the different properties of surfaces in relation to moment of inertia.

CO4: Analyse and solve different problems of kinematics and kinetics.

CO5: Analyse and solve with and without damping of SODF.

PROGRAM OUTCOMES

By the culmination of this program, the graduate acquires the following ability:

1. **Engineering Knowledge:** Apply to knowledge of Mathematics, Science and Engineering in five broad areas of Civil engineering namely Structures, Water resources, Geotechnical, Transportation and Environmental Engineering for solution of complex problems in the Civil Engineering.
2. **Problem Analysis:** Use first principle of Mathematics and Civil Engineering concepts to design and conduct experiments as well as to analyze and interpret data to analyze the complex Civil Engineering problems.
3. **Design/Development of Solutions:** to design a system, component or process to meet desired needs with respect to societal needs of public within realistic constraints.
4. **Conduct Investigations:** Use research based knowledge and research methods to identify, formulate and solve engineering problems.
5. **Modern Tool Usage:** create, select or apply appropriate engineering techniques, skills and modern engineering tools like Software necessary for Civil Engineering practice.
6. **Society and Engineer:** to understand the role and responsibility of a professional Civil Engineering in the social, health, safety and cultural issues.
7. **Environment and Sustainability:** to understand the impact of engineering solutions in a global, economic, environmental and societal context.
8. **Ethics:** to understand the professional ethics and humanitarian ethics as pertaining to norms of Civil Engineering practice.
9. **Individual and Team Work:** to function effectively as an individual and applying the principle of "UNITY IN DIVERSITY" with a spirit of teamwork.
10. **Communication:** to communicate effectively (i.e Simple, Clear and Complete) by design and drawing including use of relevant codes, writing effective technical reports and make oral or written presentation as per the need of project.
11. **Project Management and Finance:** Demonstrate knowledge and understanding of Civil Engineering and project management principles and apply them to manage/complete within the stipulated period and funds.
12. **Life-long Learning**: Recognition the need for and develop competencies necessary for life-long learning so as to offer enhanced knowledge and skill in globally changing and challenging project.

Mapping of CO'S and PO'S:

(S/M/W) indicates strength of correlation 3-strong ,2- medium ,1- weak												
CO'S	**Program Outcomes(PO'S)**											
	PO1	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**
CO1	3	3	2	2	-	-	-	-	1	-	-	-
CO2	3	3	2	2	-	-	-	-	1	-	-	1
CO3	3	2	2	2	1	-	-	-	1	-	-	1
CO4	3	3	3	3	-	-	-	-	1	-	1	-
CO5	3	3	2	3	-	-	-	-	1	-	-	-

Course Syllabus

Module 1:

Force systems and resultant of force systems, Moments and couples, Equillibrium of system of forces, Free body diagrams.

Module 2:

Friction and types of friction, limiting friction, Laws of friction, Static and Dynamic friction, Wedge friction, Screw jack.

Module 3:

Simple Truss, Method of section and method of joints,Beams and types of beams,Frames.

Module 4:

Centroid and centre of gravity,Area moment of inertia,Theorems of moment of inertia,

Module 4:

Virtual work and Energy method- Principal of virtual work,virtual displacements,Conservative forces and potential energy(elastic and gravitional),Energy equation for equilibrium.

Module 6:

Rectilinear motion,Plane of curvilinear motion,Work kinetic energy,Power,Potential energy, Impulse momentum.Impact.

Module 7:

D' Alembert principle and its applications in plane motion, Work energy principle and its application,Kinetics of rigid body rotation.

Theory: 42 hours

References:

1. **S S Bhavikati, Engineering Mechanics,New age publication ,New Delhi.**
2. A Nelson ,Engineering Mechanics Statics and Dynamics.
3. Beer and Johnston. Engineering Mechanics.
4. R S Khurmi, A Text book of Engineering Mechanics.

Course Assessment Methods:

1. Mid Semester Test
2. Assignments
3. End Semester Exam.

CHAPTER

1

Coplaner Forces

Introduction :

In this chapter we shall learn the procedure of adding the forces, resolving the forces and projecting the forces. We will begin our study by defining a force and restrict our study of forces contained in a single plane.

1.1. Definition of Force:

It is defined as an external agency which produces or tends to produce, destroys ~~an~~ or tends to destroys the motion.

Force is a vector quantity and S.I. unit is Newton.

1 Newton force is defined as force required to produce unit acceleration on unit mass.

$$\therefore \boxed{1\ Kg = 9.81\ N}$$

1.2. Characteristics of Force:

A force is characterised by following properties.

1) Magnitude
2) Direction
3) Nature or Sense
4) point of application.

Now, we shall discuss each property in detail.

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1) Magnitude : This represents the value of force i.e 5 KN, 9 KN etc.

2) Direction : Force is represented by line of action and the angle it forms with some fixed axis.

3) Nature or Sense : The nature of force is represented by arrowhead. Generally, it is termed as push or pull.

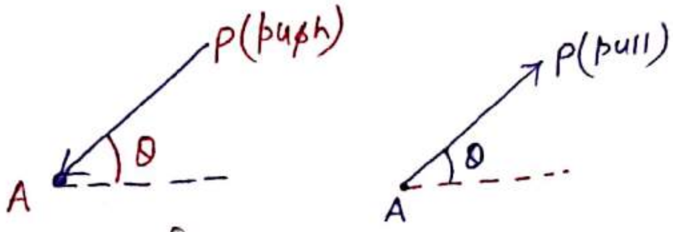


Fig:

4) point of application : It is the location of a point on a body where force is acting.

For the force shown in Fig. ...., magnitude of force is 5 KN, direction is 45° with the horizontal in fourth quadrant, point of application is C and line of action is AB in pull form.

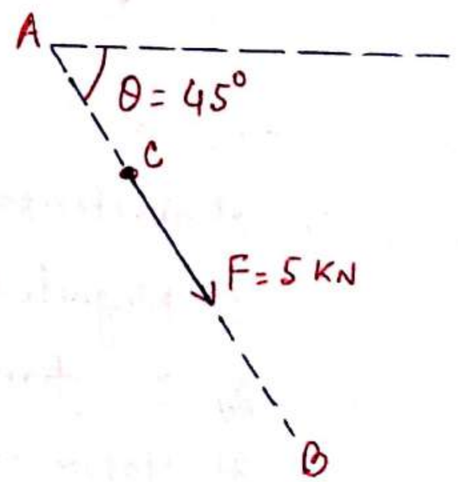


Fig:

1.3 Representation of a Force :

A force when acting on a particle or a body may be representated by

1) Vector representation.

2) Bow's notation.

Now, we shall discuss each in detail.

1) **Vector representation :** In this method, force is graphically represented by a straight line drawn parallel to the line of action of the force on any suitable scale.

The length of line represents magnitude and arrow indicates the direction of force.

2) **Bow's notation :** In this method, a force is represented by

(a) by placing a single letter (say P, Q, R----) and an arrow to indicate direction.

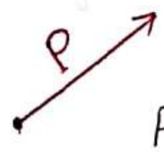
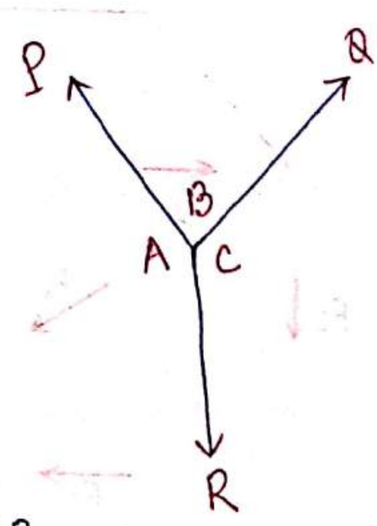


Fig :

(b) by putting an alphabet letter between two forces.



Force P is designated by AB.

Fig :

1.4 System of Forces :

When a single agency is acting on a particle it is called as force but when number of forces simultaneously acting the system so formed is called as system of forces.

Types of system of forces:

There are ~~mainly~~ following types of system of forces.

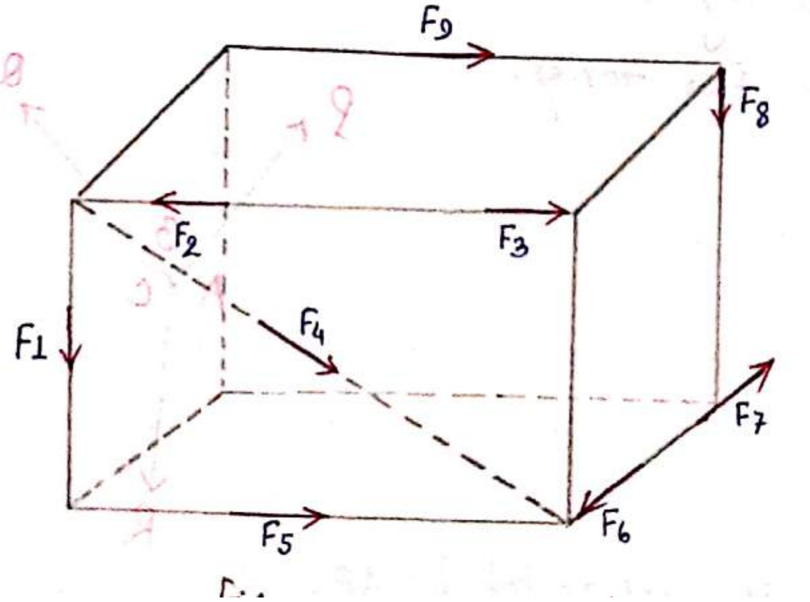
1) Co-planer forces
2) Non-coplaner forces
3) Collinear forces
4) Non-Collinear forces
5) Concurrent forces
6) Non-Concurrent forces
7) Parallel forces : a) Like parallel forces (b) Unlike parallel forces

Now, we shall discuss each systems in detail.

1) Co-planer Force System :

The forces which are acting in the same plane are known as Co-planer forces or Co-planer force system.

e.g. In Fig. -- , forces F_2, F_3 and F_9 are Coplaner.



2) Non- coplaner Force System :

A force system in which the forces acting in different planes is called as Non- coplaner force system.

Non- coplaner forces are also called as space forces or spatial force system.

e.g. In fig--, forces F_1 and F_9 are non- coplaner forces.

3) Collinear forces :

The forces which are acting along the same straight line are called as Collinear forces.

e.g. In fig.---, forces F_2 and F_3 are collinear forces.

4) Non- collinear Forces :

The forces which are not acting along a straight line are called as Non- collinear forces.

e.g. In fig.---, forces F_2 and F_9 are non-collinear forces.

5). Concurrent Forces :

The forces which are passing through a common point are called concurrent forces.

e.g. In fig--, forces F_1 and F_2 are concurrent forces.

6) Non-Concurrent forces :

The forces which are not passing through a common point are called as non-concurrent forces.

e.g. In fig---, forces F_3 and F_8 are non-concurrent forces.

7) Parallel Forces :

The forces whose lines of action are parallel to each other are known as parallel forces.

There are two types of parallel force system.

a) Like parallel Forces : The forces which are parallel to each other and ~~having~~ same direction are called as like parallel forces.

e.g. In fig---, forces F_3 and F_9 are like parallel forces.

(b) Unlike parallel Forces : The forces which are parallel to each other and ~~having~~ different directions are called as unlike parallel forces.

e.g. In fig---, forces F_2 and F_9 are unlike parallel forces.

Note : It is not necessary that each system exists in isolation but combination of systems may practically exist.

1.5 Composition and Resolution of Forces :

(a) Composition of forces : The process by which the single resultant force is found out, known as composition of forces.

(b) Resolution of forces : The process by which a given force is split up into two or more components without changing the effect of the same.

Generally a force is to be resolved into two perpendicular or non-perpendicular components but a force can be resolved into number of components.

1.6 Define Resultant and Equillibrant :

* Resultant : It is a single force which produces the same effect that as produced by number of forces when acting simultaneously. In fact, resultant force replaces the number of forces. It is denoted by (R).

* Equilibrant : It is a single force which when acting with all other forces keeps the body at rest or in equilibrium. It is denoted by (E).

1.7 Relation between Resultant (R) and Equilibrant (E)

The resultant and Equilibrant are equal in magnitude but opposite in direction.

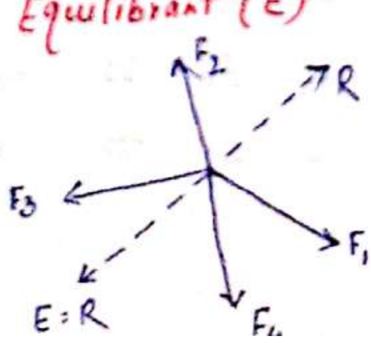


Fig:

1.8 Methods of composition: (To find R):

There are different methods to find out resultant of different force systems.

1.8.1. Resultant of two concurrent Forces:

"Law of parallelogram of forces": (Analytical approach)

It states " Two forces acting simultaneously on a body, if represented in magnitude and direction by the two adjacent sides of a parallelogram, then diagonal of a parallelogram, from the point of intersection of above two forces, represents the resultant force in magnitude and direction".

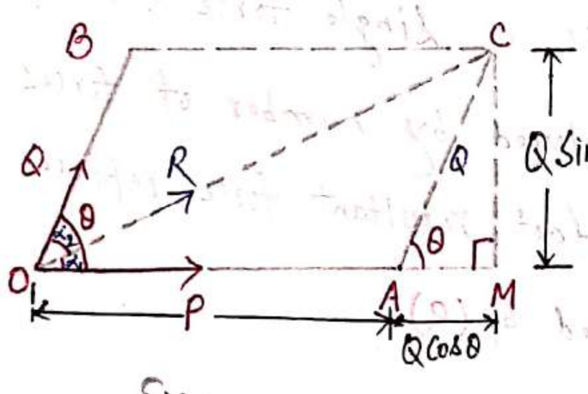


Fig:

Let two forces P and Q acting at a point O represented by two sides OA and OB of a parallelogram OACB.

Let θ be the angle between two forces P and Q.

α_1 be the angle between P and R.

α_2 be the angle between Q and R.

Now, drop a perpendicular CM and produce OA

In Δ CMA, we have $CM = Q\sin\theta$ and $AM = Q\cos\theta$.

Magnitude of R:

In Δ OMC,

We have $$(OC)^2 = (OM)^2 + (CM)^2$$

$$(OC)^2 = (OA + AM)^2 + (CM)^2$$

$$\therefore R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$\therefore R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$\therefore R^2 = P^2 + 2PQ\cos\theta + Q^2(\cos^2\theta + \sin^2\theta)$$

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \text{... Magnitude}$$

Direction of resultant (R):

In Δ OMC,

We have $$\tan\alpha_1 = \frac{CM}{OM} = \frac{CM}{OA + AM}$$

$$\therefore \tan\alpha_1 = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{... Direction.}$$

or, Similarly,

$$\tan\alpha_2 = \frac{P\sin\theta}{Q + P\cos\theta} \quad \text{..... Direction.}$$

Particular cases :

a) When two forces P and Q are acting in same direction we have $\theta = 0^\circ$.

$\therefore$ Magnitude of resultant,

$$R = (P + Q)$$

and ~~direction~~ of will act in the same direction of force.

b) When two forces P and Q are perpendicular. We have $\theta = 90^\circ$.

$\therefore$ Magnitude of resultant,

$$R = \sqrt{P^2 + Q^2}$$

And direction $\tan \alpha_1 = Q/P$.

c) When two forces P and Q are acting in opposite direction, we have $\theta = 180^\circ$.

$\therefore$ Magnitude of resultant,

$$R = (P - Q) \text{ or } (Q - P)$$

And it will act in the direction of bigger force out of the two.

1.8.2. Resultant of Two or More Forces :

Method of resolution : When two or more Co-planer Concurrent or non-Concurrent forces acting on a body, the resultant can be found out by using resolution procedure.

Magnitude of resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

and Direction $\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$

Where ΣF_x = Algebric sum of all x-components

ΣF_y = Algebric sum of all y-components.

θ = Angle of R with x-axis.

1.9. Resolution of a Force :

There are following methods of resolving a force.

1) Orthogonal (perpendicular) resolution

2) Non-perpendicular resolution

3) Resolution into two parallel Components.

Now, we will discuss each ~~prod~~ resolution procedure.

1.9.1. Orthogonal (Perpendicular) Resolution :

In this method, generally a given force is split up into two mutually perpendicular components preferably

a) Horizontal or X- Component

b) Vertical or Y- Component.

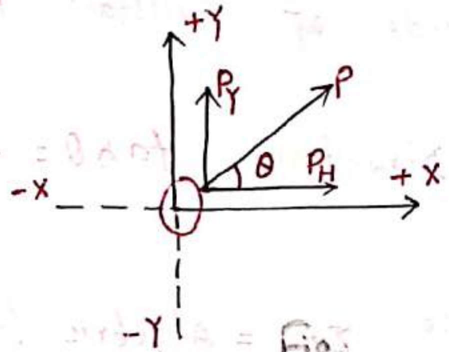


Fig:

a) Horizontal component,

$P_H = P.\cos\theta$ (angle with horizontal)

$\therefore \boxed{P_H = P\cos\theta}$

b) Vertical component,

$P_V = P.\sin$ (angle with horizontal)

$\boxed{P_V = P.\sin\theta}$

Special cases of resolution :

a) Selecting x-axis along the plane and y-axis perpendicular to plane

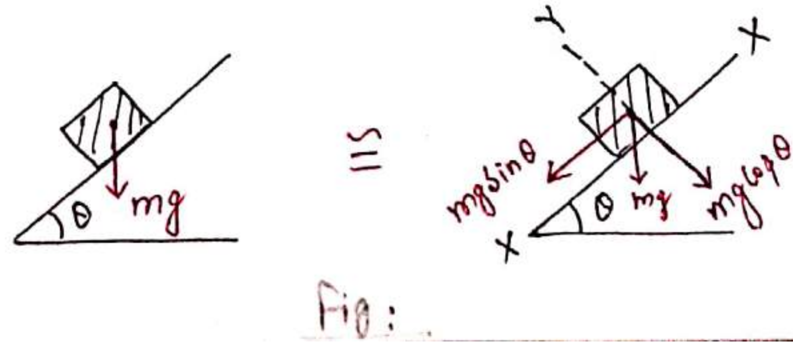


Fig :

1) Component along the plane $= mg\cos(90-\theta)$

$= mg\sin\theta$

2) Component ~~along~~ perpendicular to the plane

$= mg\sin(90-\theta)$

$= mg\cos\theta$

b) Here force 'p' can be resolved in following manners. (Ref. Fig... and ..).

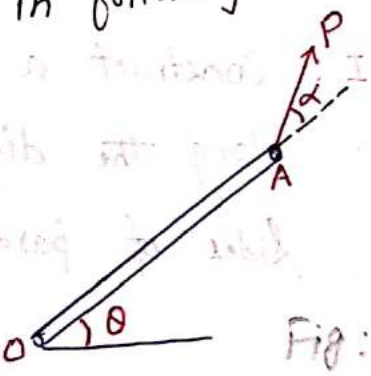


Fig:

1) Along and perpendicular to OA. (Ref. Fig..).

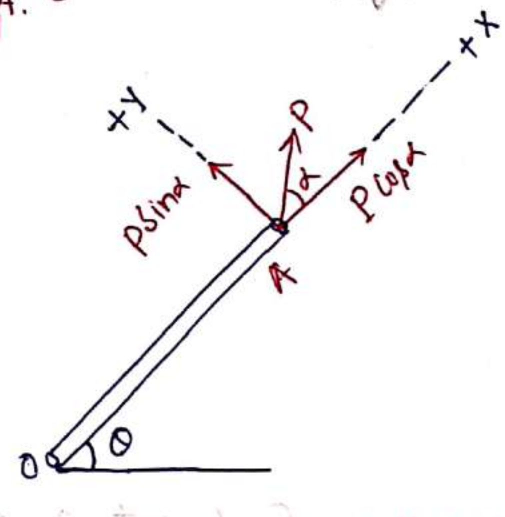


Fig...

2) Horizontal and Vertical components (Ref. Fig...)

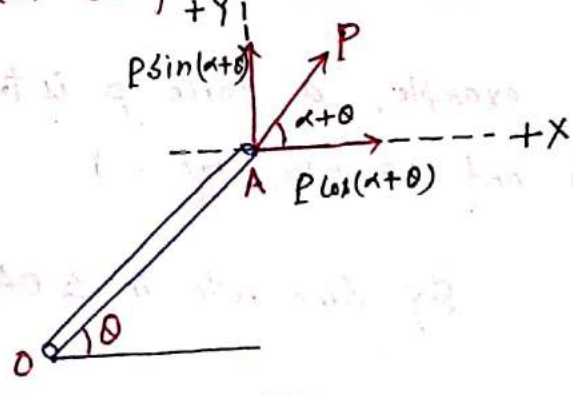


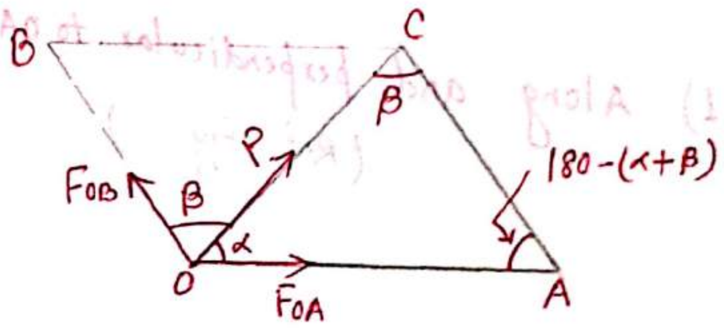
Fig...

1.9.2. Non-perpendicular components :

When a force is required to resolved in to two directions which are not perpendicular to each other, the resolution is called Non-perpendicular resolution.

Following is the simple procedure to resolve a force into two non-perpendicular directions.

Step I : Construct a parallelogram by keeping original given force (P) along the diagonal and two components along two adjacent sides of parallelogram (passing through same point).



$\alpha+\beta \neq 90^\circ$

Fig :

Step II : Find out 3 angles of any one triangle.

Step III : Apply sine rule in that triangle.

For example, a force P is to be required to resolve along directions OA and OB (Ref. Fig ---).

By sine rule in Δ OAC,

$$\frac{F_{OB}}{\sin\alpha} = \frac{F_{OA}}{\sin\beta} = \frac{P}{\sin[180-(\alpha+\beta)]}$$

$$\therefore \quad F_{OA} = \frac{P \cdot \sin\beta}{\sin[180-(\alpha+\beta)]}$$ ... (Component along OA)

$$F_{OB} = \frac{P \cdot \sin\alpha}{\sin[180-(\alpha+\beta)]}$$ ... (Component along OB).

1.10. Moment :

The turning effect produced by a force on the body is called moment of the force.

The moment of a force about any point is the product of Magnitude of the force and perpendicular distance between the line of action of force and the axis of rotation.

The point about which moment is taken is called moment center.

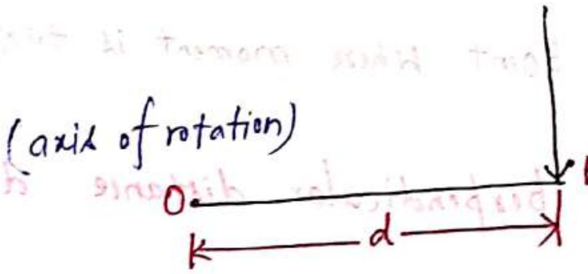


Fig :

$\therefore$ Moment of force P about point O

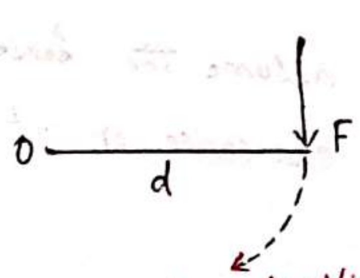
i.e $M_0 = F \times d$

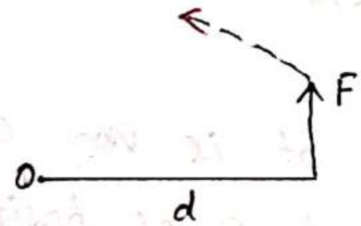
Moment is zero when either $F = 0$ or $d = 0$.

The S.I. unit of moment is N-m or KN·m.

Sign - Conventions : The moment is the rotational effect produced by a force. There are two types of rotation

1) Clockwise 2) Anticlockwise.





Clockwise moment $\rightarrow$ positive

Anticlockwise moment $\rightarrow$ Negative

We shall assume this sign conventions.

Varignon's theorem (Law of Moments):

1.11 ~~Graphical Representation of Moment:~~

Statement : The algebric sum of moments of all the forces about any point is equal to the moment of their resultant about the same point."

Mathematically,

$$R \cdot d = \Sigma M$$

Where d = perpendicular distance of Resultant 'R' about a point where moment is taken.

$\therefore$ perpendicular distance $d = \dfrac{\Sigma M}{R}$

Application : By using Varignon's theorem, we can find the horizontal and vertical distances of resultant.

Horizontal distance $x = \dfrac{\Sigma M}{\Sigma F_y (= \Sigma V)}$

Vertical distance $y = \dfrac{\Sigma M}{\Sigma F_x (= \Sigma H)}$

Use : Varignon's theorem of moments (V.T.M) is useful to find out position or location of resultant of non-concurrent force-system.

Note : It is very convenient to assume the sense of moment of R as positive (whether clockwise or anticlockwise).

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Proof : We will prove Varignon's theorem by considering two concurrent forces P and Q having resultant R.

Let P and Q be the concurrent forces acting at O.

Let R be the resultant of P and Q

Let P, Q and R make angles θ, β and α with horizontal axis respectively.

- Now, select a convenient moment center 'A'. (Ref. Fig ---).

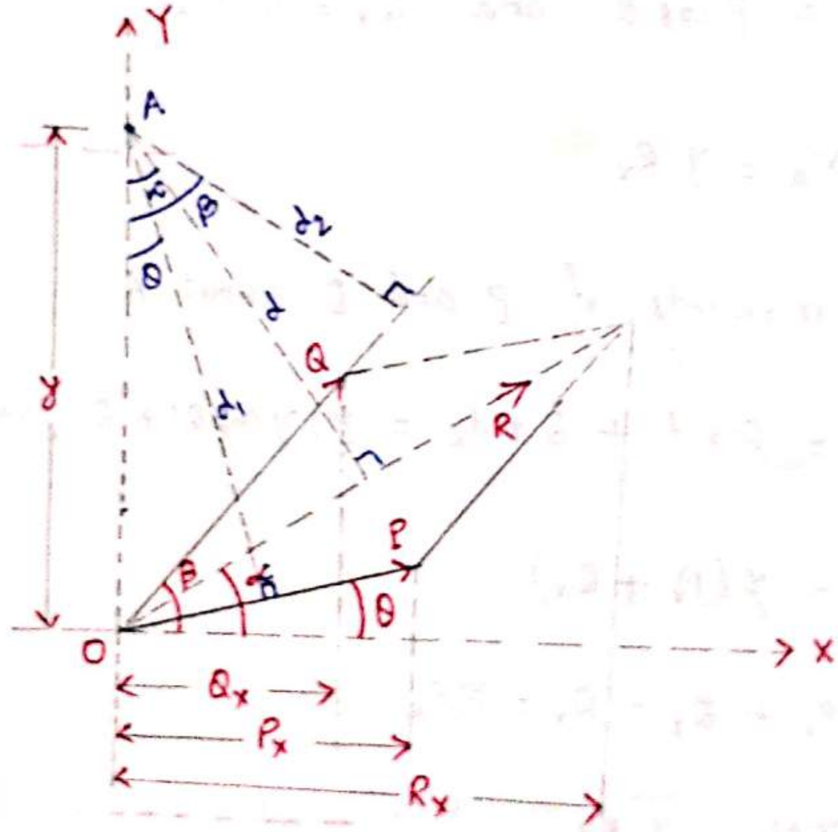

Fig.

From A, perpendicular distances on the lines of action of P, Q and R are d_1, d_2 and d respectively. Components of P, Q and R in X direction are P_x, Q_x and R_x.

$\therefore$ Moment of resultant about A will be

$$M_A = R \times d \quad \text{------- Anticlockwise}$$

But from geometry, we have

$$d = y\cos\alpha \; ; \quad d_1 = y\cos\theta \text{ and } d_2 = y\cos\beta.$$

$$\therefore M_A = y\cos\alpha \, R$$

Now; $R_x = \Sigma F_x = R\cos\alpha$

$P_x = P\cos\theta$ and $Q_x = Q\cos\beta$.

$$\therefore M_A = y \cdot R_x \quad \text{------ (i)}$$

Consider moments of P and Q about A.

$$\therefore \Sigma M_A = P \times d_1 + Q \times d_2 = P(y\cos\theta) + Q(y\cos\beta) \text{ --- (Anticlockwise)}$$

$$\therefore \Sigma M_A = y(P_x + Q_x)$$

But $P_x + Q_x = R_x = \Sigma F_x$

$$\therefore \Sigma M_A = y \cdot R_x \quad \text{------ (ii)}$$

From (i) and (ii), it is clear that Moment of resultant at A is equal to total moments of P and Q about A.

Thus Varignon's theorem is proved by equations (i) and (ii).

1.12. Couple :

Two equal, opposite and parallel forces having different line of action are said to form couple.

The distance between two forces is known as arm or lever of the couple.

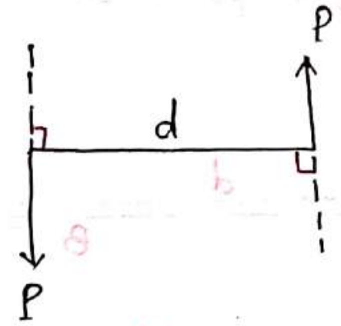

Fig :

The moment of a couple,

i.e $M = P \times d$

Notes : 1) The resultant of a couple is zero.

2) The moment of couple is independent of the moment center.

3) The effect of a couple is unchanged if

a) The couple is shifted to any other position in its plane.

(b) The couple is rotated through any angle in its plane.

4) The couple can be balanced by another couple of opposite nature.

1.13. Repacing a Force by a Force-Couple System:

When a force is required to transfer from point A to point B, We can transfer the force directly without changing its magnitude and direction but along with the moment of force about point B.

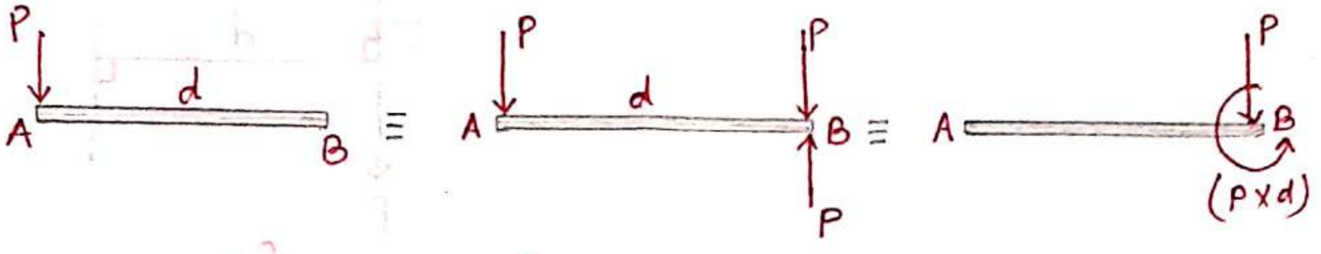

Fig :

As a result of parallel transfer, a system is obtained which is always a combination of a force and a moment or couple. This system which is consisting a force and a couple at a point is known as Force-Couple system.

Fig--- (a) shows a bar subjected to a force P(↓) at A. Now when this force is required to be transferred from A to B. Let us introduce a system of forces at B which is in equilibrium. (Ref. Fig--(b)). The force system in Fig-- (b) can be reduced in a system subjected to a downward force at B and an anticlockwise couple at B. (Ref. Fig--- (c)) which is a force-couple system.

Problems Based on Components

EX. 1.1: Find orthogonal components for following force system.

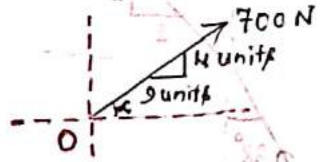

Fig: Ex. 1.1

Soln : Here angle of force with horizontal

$$\tan\alpha = \frac{4}{9}$$

$$\therefore \alpha = 23.96^\circ$$

$$\therefore x \text{ Component} = 700 \cos 23.96^\circ = 639.67 \text{ N} (\rightarrow)$$

$$y \text{ component} = 700 \sin 23.96^\circ = 284.27 \text{ N} (\uparrow)$$

EX. 1.2: Find x and y components for following force System

(a) x Component $= -7 \cos 35^\circ = -5.73$ KN
y Component $= +7 \sin 35^\circ = 4.01$ KN

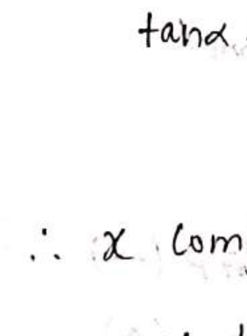

Fig. Ex. 1.2(a)

(b) x Component $= -11 \cos 43^\circ = -8.04$ KN
y Component $= -11 \sin 43^\circ = -7.50$ KN

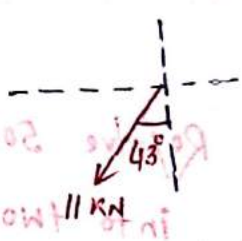

Fig: Ex. 1.2(b)

(c) x Component $= +15 \cos 55^\circ = 8.60$ KN
y Component $= -15 \sin 55^\circ = -12.29$ KN

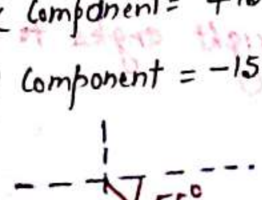

Fig: Ex. 1.2(c)

(d) x Component $= -5 \cos 50^\circ = -3.21$ KN
y Component $= +5 \sin 50^\circ = +3.83$ KN

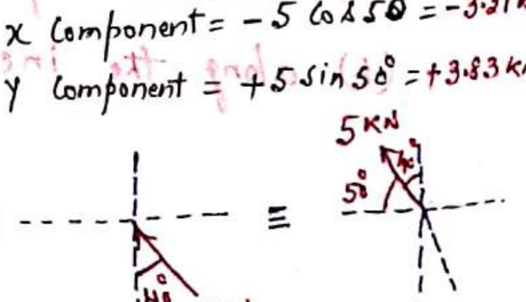

Fig: Ex 1.2(d)

Ex 1.3: Find Components of forces P and Q in horizontal and vertical directions.

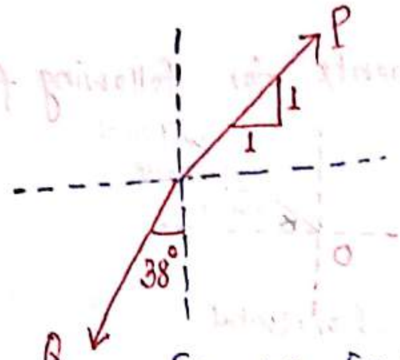

Fig: Ex. 1.3

Soln: Components of force P

Here angle of force P with horizontal

$$\tan\alpha = \frac{1}{1} \Rightarrow \alpha = 45^\circ$$

$\therefore$ x component $= +P\cos 45^\circ = 0.707P\ (\rightarrow)$

y component $= +P\sin 45^\circ = 0.707P\ (\uparrow)$

Components of force Q

$\therefore$ x component $= -Q\sin 38^\circ = 0.615\ Q\ (\leftarrow)$

y component $= -Q\cos 38^\circ = 0.788\ Q\ (\downarrow)$

Ex. 1.4: Resolve 500 N force acting on a block as shown in Fig.... into two Components as given below:

(a) horizontal component and vertical component

(b) along the inclined plane and at right angles to the plane.

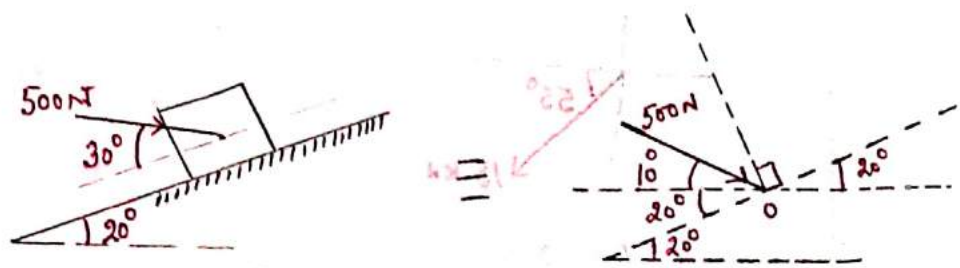

Fig: Ex. 1.4

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Soln : (a) Component of force 500 N along Horizontal and Vertical plane

Angle made by 500 N force with horizontal = 10° (Ref. fig Ex 1.4)

$\therefore$ X- Component (F_x) = $+500 \cos 10^\circ = 492.40\ N\ (\rightarrow)$

Y- Component (F_y) = $-500 \sin 10^\circ = -86.82\ N = 86.82\ N\ (\downarrow)$

(b) Component of force 500 N along inclined plane and at right angle to the plane.

Angle made by 500 N force with plane = 30° (Ref. fig Ex 1.4)

$\therefore$ Component along the plane $(F_t) = +500 \cos 30^\circ = 433.01\ N\ (\angle 20^\circ)$

Component at right angles to the plane $(F_n) = -500 \sin 30^\circ = -250\ N$

$= 250\ N\ (70^\circ)$

Ex 1.5 Resolve 400 N force acting on a block as shown in Fig. --- into two components, along the inclined plane and at right angles to the plane.

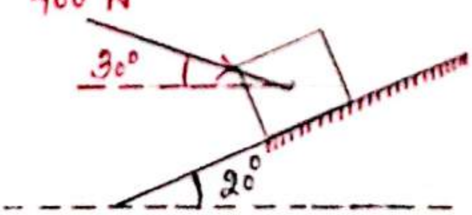

Fig: Ex 1.5

Soln : Component of force 400 N along the inclined plane and at right angles to the plane.

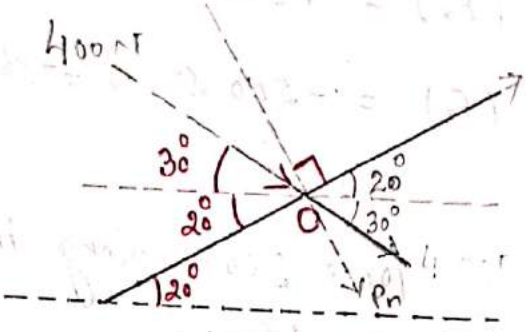

Fig : Ex 1.5(a)

Angle made by 400 N force with plane = 50°

$\therefore$ Component along the plane $(P_t) = +400 \cos 50^\circ = 257.11$ N (20°)

Component along the at right angle to the plane $(P_n) = -400 \sin 50^\circ$

$= -306.41$ N

$= 306.41$ N (70°)

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Ex 1.6 : Resolve force of 50 KN applied at end B of a rigid pole AB (which is perpendicular to the sloping ground) into following components at right angles to each other:

a) Along the pole and at right angles to the pole

b) Along horizontal and vertical component

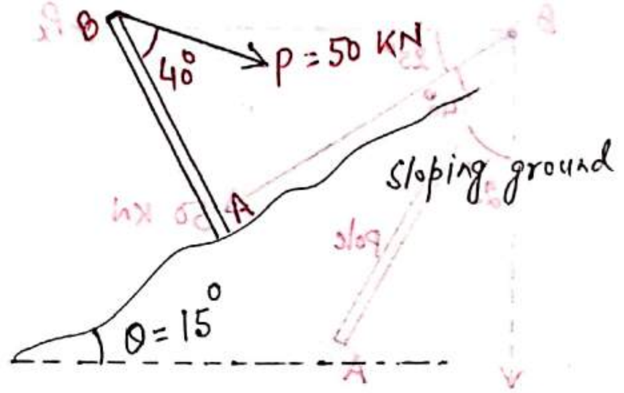

Fig: Ex 1.6

Soln : (a) Component of force 50 KN along the pole and at right angles to the pole.

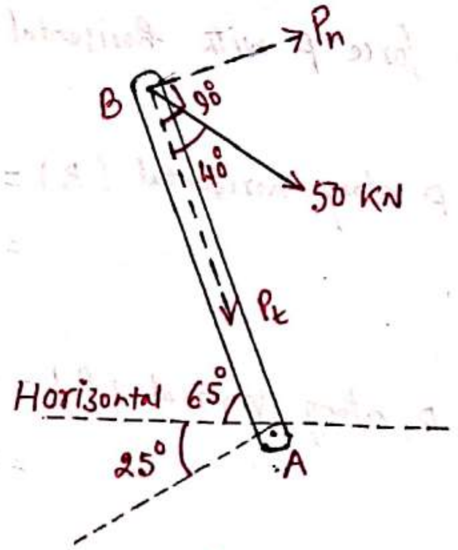

Fig: Ex 1.6(a)

$\therefore$ Component along the pole $(P_t) = -50 \cos 40^\circ = -38.30$ KN

$= 38.30$ KN ($\searrow 65^\circ$)

(b) Component of force 50 KN along horizontal and vertical ~~component~~ plane.

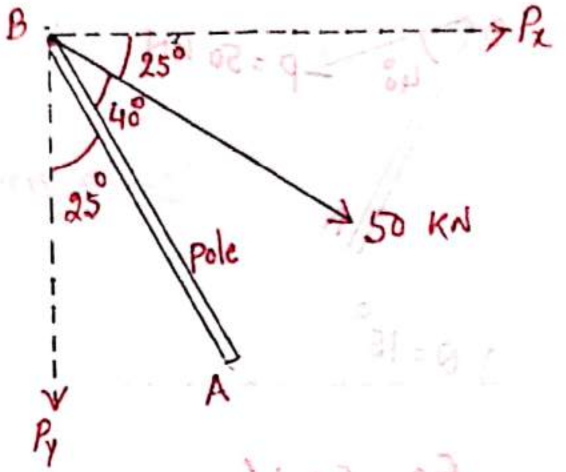

Fig: Ex 1.6(b)

Now, angle made by pole with horizontal = 65^0, and

angle made by force P with horizontal = 25^0.

$\therefore$ Component of force P along horizontal $(P_x) = +50 \cos 25^0$

$= 45.31$ KN $(\rightarrow)$

Component of force P along vertical $(P_y) = -50 \sin 25^0$

$= -21.13$ KN $= 21.13$ KN $(\downarrow)$

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Problems based on Coplanar concurrent forces

Followings are the basic steps to find the resultant of coplanar concurrent forces:

1. Rearrange, if necessary, all the forces in either pull form or push form.
   Compute angle of given forces with horizontal or vertical, measured in anticlockwise sense.

2. Find ΣH.
   i.e algebric sum of all the horizontal components of all forces. Note the sign i.e (+ve) or (-ve).

3. find ΣV.
   i.e algebric sum of all the vertical components of all forces. Note the sign i.e (+ve) or (-ve).

4. find the magnitude of resultant R, by
   i.e $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

5. (a) find the angle (θ) of the resultant R with the horizontal by the equation i.e $\cancel{\tan}\,\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

   (b) find the angle $\cancel{(\phi)}$ ($\phi = 90^\circ - \theta$) of the resultant R with the vertical by
   i.e $\phi = \tan^{-1}\left(\frac{\Sigma H}{\Sigma V}\right)$.

Ex. 1.13. Find the resultant of the following force system as shown in figure. Ex 1.13.

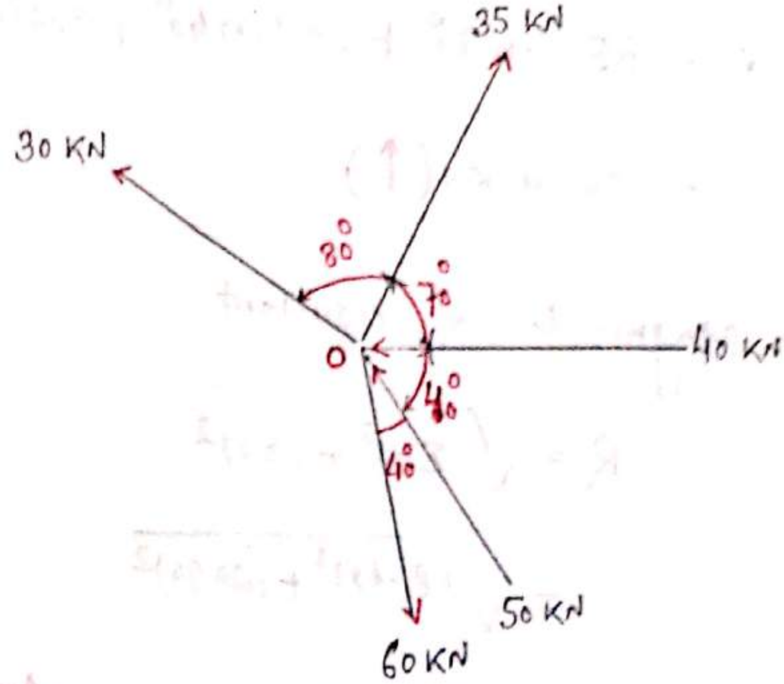


Fig: Ex 1.13

Soln: Rearranging all forces 'awaygoing' from their point of application, O (using principles of transmissibility of forces).

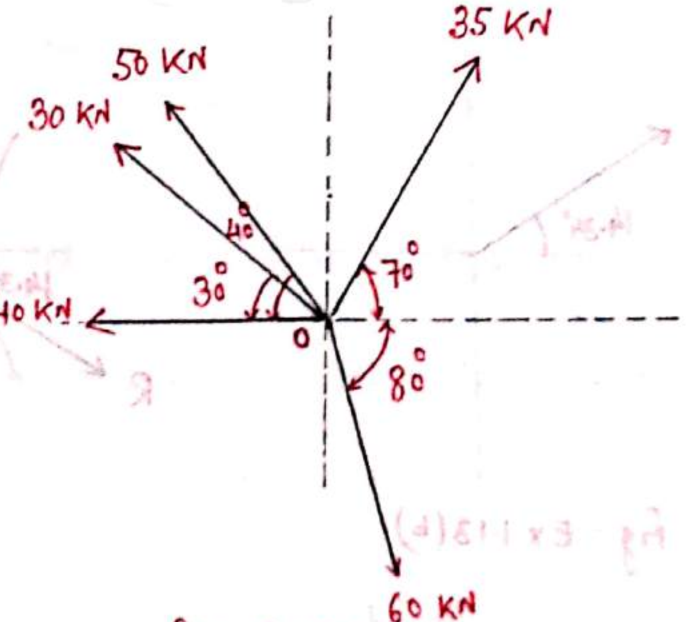


Fig: Ex 1.13(a)

Resolving the forces along x and Y axis

$$\underset{\xrightarrow{+}}{\Sigma H} = 35 \cos 70^\circ - 50 \cos 40^\circ - 30 \cos 30^\circ - 40 + 60 \cos 80^\circ$$

$$= -81.89 \text{ KN} = 81.89 \text{ KN} (\leftarrow)$$

$$+\uparrow \Sigma V = 35 \sin 70^\circ + 50 \sin 40^\circ + 30 \sin 30^\circ - 60 \sin 80^\circ$$

$$= 20.94 \text{ KN} (\uparrow)$$

$\therefore$ Magnitude of resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(81.89)^2 + (20.94)^2}$$

$$= 84.52 \text{ KN} \quad \text{---- Ans.}$$

Direction, $\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$$\therefore \theta = \tan^{-1}\left[\frac{20.94}{81.89}\right] = 14.34^\circ \quad \text{---- Ans.}$$

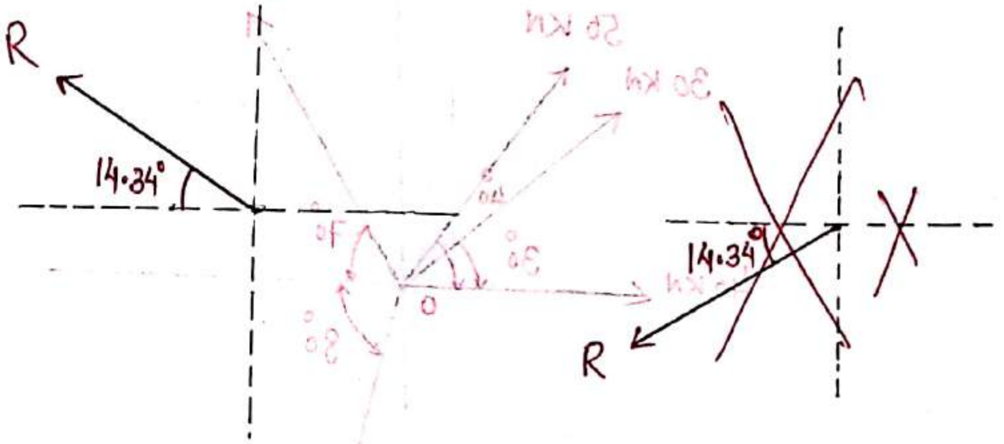


Fig : Ex 1.13(b)

R is acting in IInd quadrant.

Ex. 1.14 : At a point on a body, four forces act as given below. Determine resultant and its orientation with respect to north direction. (All forces are 'away going').

i) 800 N due east

ii) 500 N in north west

iii) 1200 N at 35° south of west

iv) 600 N at 30° east of north.

Soln: Arrange and draw sketch of the given forces (Ref: fig. Ex 1.14)

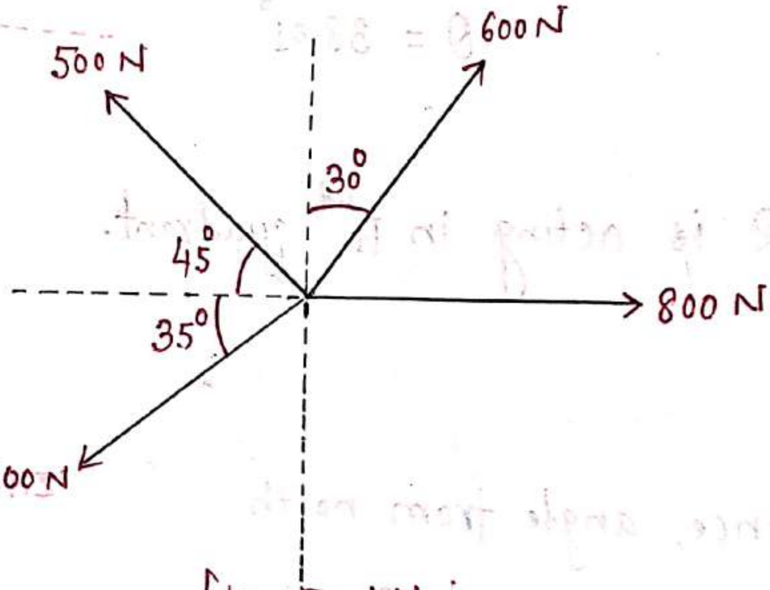


Fig: Ex. 1.14

Resolving forces along x and y-axis.

$$\underset{+}{\xrightarrow{\Sigma H}} = 800 + 600 \sin 30^\circ - 500 \cos 45^\circ - 1200 \cos 35^\circ$$

$$= -236.53 \text{ N} = 236.53 \text{ N} (\leftarrow)$$

$$+\uparrow \Sigma V = 600 \cos 30^\circ + 500 \sin 45^\circ - 1200 \sin 35^\circ$$

$$= 184.87 \text{ N} (\uparrow)$$

$\therefore$ magnitude of resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(236.53)^2 + (184.87)^2} = 300.20 \text{ N} \quad --- \text{ Ans.}$$

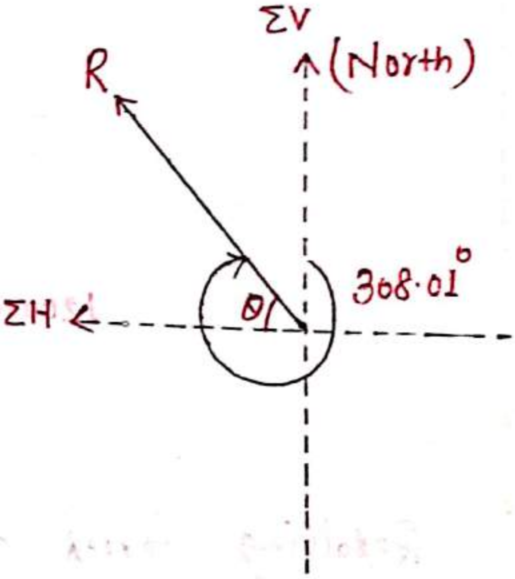
Direction:

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\therefore \theta = \tan^{-1}\left[\frac{184.87}{236.53}\right]$$

$$\theta = 38.01^\circ \quad ---- \text{ Ans.}$$

R is acting in IInd quadrant.



Hence, angle from north

$$= (270^\circ + \theta)$$

$$= 308.01^\circ \text{ (clockwise)}$$

fig: Ex. 1.14(a)

Problems based on Coplanar Non concurrent System

Following procedure is used to determine resultant and the line of action of resultant of Coplanar Non-Concurrent System

1. Resolve all forces into components along X and Y directions assuming X and Y axis are passing through the origin of reference point.

2. Find ΣH and ΣV

   $\therefore$ magnitude of resultant $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

   Direction, $\tan\theta = \left[\frac{\Sigma V}{\Sigma H}\right]$

3. To find the line of action of resultant R at a distance d from reference point / origin.

   using Varignon's theorem of moment

   i.e $R \times d = \Sigma M$

   where ΣM is the moment about reference point / origin due to force (including given moment, if any).

4. Mark the line of action of resultant 'R' at a distance 'd' i.e $d = \frac{\Sigma M}{R}$ from reference point / origin along the perpendicular line.

Line of action of resultant R is determined from the nature of moment at reference point / origin (i.e. clockwise or anticlockwise).

5. Using X-component ΣH and Y-component ΣV find horizontal and vertical distance of resultant

i.e. Horizontal distance $\bar{X} = \frac{\Sigma M}{\Sigma V}$

Vertical distance $\bar{Y} = \frac{\Sigma M}{\Sigma H}$.

Ex. 1.22. Determine the resultant of the coplanar non-concurrent force system as shown in fig. Ex 1.22. Calculate its magnitude, direction and locate its position with respect to the sides AB and AD.

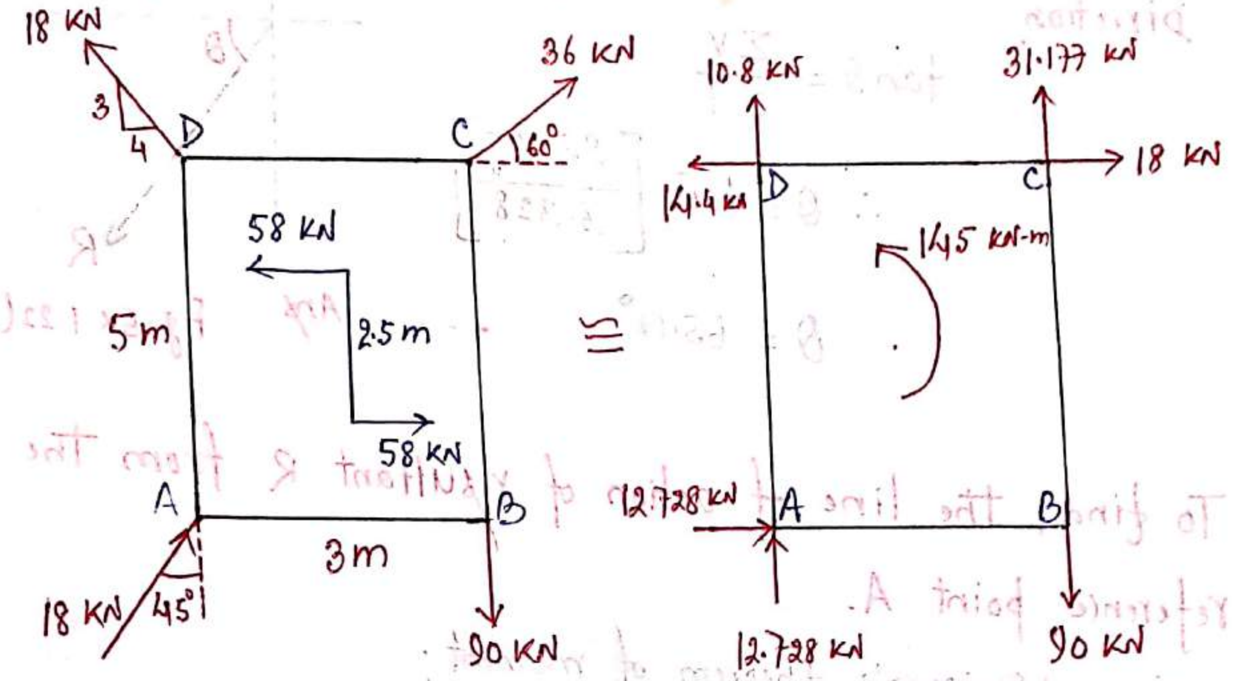


Fig : Ex 1.22 　　　　fig : Ex 1.22 (a)

Soln :

Let A be the origin.

Resolving forces along X and Y directions.

i.e $\underset{+}{\xrightarrow{\Sigma H}} = 12.728 + 18 - 14.4 = 16.328 \text{ KN} (\rightarrow)$

$+\uparrow \Sigma V = 12.728 + 10.8 + 31.177 - 90 = -35.295 \text{ KN} = 35.295 \text{ KN} (\downarrow)$

Magnitude of resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\therefore R = \sqrt{(16.328)^2 + (35.295)^2}$$

$$R = 38.889 \text{ kN} \quad \text{--- Ans}$$

Direction:

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\therefore \theta = \tan^{-1}\left[\frac{35.295}{16.328}\right]$$

$$\theta = 65.17^\circ \quad \text{--- Ans}$$

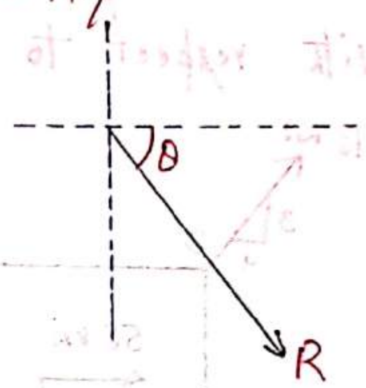


Fig. Ex 1.22(b)

To find, the line of action of resultant R from the reference point A.

Using Varignon's theorem of moment;

$$R \times d = \Sigma M_A$$

$$= 90\times3 + 18\times5 - 31.177\times3 - 14.4\times5 - 145$$

$$= 49.469 \text{ kN-m}$$

$$\therefore \text{perpendicular distance } d = \frac{\Sigma M_A}{R} = \frac{49.469}{38.889}$$

$$\therefore d = 1.272 \text{ m from A} \quad \text{--- Ans.}$$

Now, Horizontal distance (i.e along side AB)

i.e $$\bar{X}_A = \frac{\Sigma M}{\Sigma V}$$

$$\therefore \quad \overline{X_A} = \frac{49.467}{35.295} = 1.402 \text{ m}$$

Vertical distance (i.e along side AD)

i.e $$\overline{Y_A} = \frac{\Sigma M}{\Sigma H}$$

$$\therefore \overline{Y_A} = \frac{49.469}{16.328} = 3.03 \text{ m}.$$

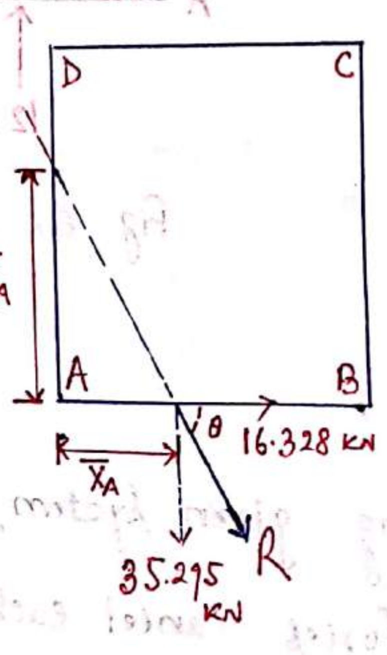


Fig: Ex 1.22 (c)

fig. Ex 1.23. The system is coplanar. Determine, resultant of the system with reference to point A. Also replace this system by a system of three forces acting along the sides of the triangle.

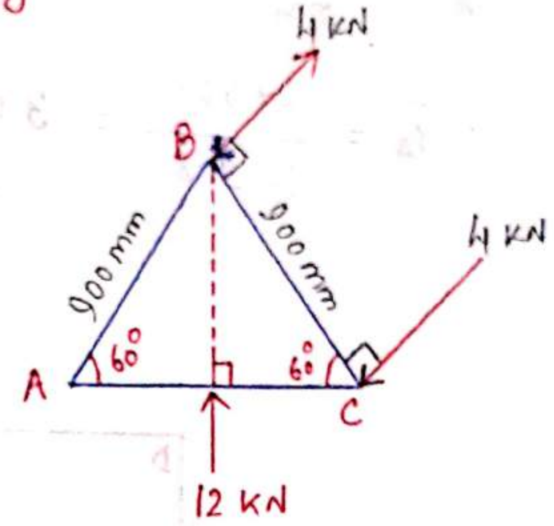


Fig: Ex. 1.23

Soln:

By observing given system, we find that the components of 4 KN forces cancel each other.

ie they form a clockwise couple of $(4 \times 0.9) = 3.6$ KN-m.

Now, $+\uparrow \Sigma V = 12$ KN

∴ magnitude of resultant & direction

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{0 + 12^2} = 12 \text{ KN} (\uparrow) \quad \text{---- Ans}$$

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To find out the position of R about point A.

Using Varignon's theorem of moments.

Let d be the distance where resultant R intersects line AC (i.e. horizontal distance)

$$\therefore \ R \times d = \Sigma M_A \ (+\circlearrowleft)$$

$$= (4 \times 0.9) - (12 \times .45)$$

$$= -1.80 \text{ KN-m} = 1.80 \text{ KN-m} \circlearrowright$$

$$\therefore \ d = \frac{1.80}{12} = 0.15 \text{ m} \quad \text{---- Ans}$$

$\therefore$ d = 0.15 m (= 150 mm) to the right of A.

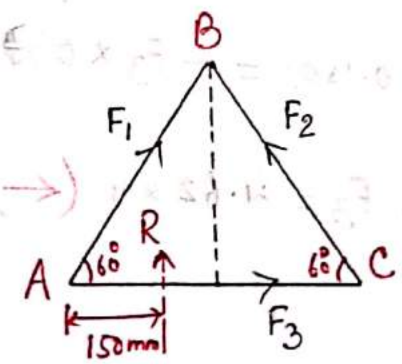


Fig: Ex. 1.23(a)

Now, Assume new system of three forces as required consisting of F_1, F_2 and F_3. (Ref. Fig. Ex. 1.23(a))

For equilateral triangle,

$$\text{altitude} = \sqrt{.90^2 - .45^2}$$

$$= 0.779 \text{ m}$$

using V.T.M. about point A;

$$R \times d = \Sigma M_A$$

$$\therefore 12 \times 0.150 = F_2 \sin 60^\circ \times 0.900$$

.... other forces and components pass through A.

$$\therefore F_2 = 2.31 \text{ KN} \quad 60^\circ$$ --- Ans.

Again, using V.T.M. about point C;

$$12 \times (0.900 - 0.150) = F_1 \sin 60^\circ \times 0.900$$

$$\therefore F_1 = 11.55 \text{ KN} \quad 60^\circ$$ ---- Ans

using V.T.M. about point B;

$$12 \times (0.450 - 0.150) = -F_3 \times 0.779$$

$$\therefore F_3 = 4.62 \text{ KN } (\leftarrow)$$ ---- Ans

Check :

$$\overset{+}{\rightarrow} \Sigma H = F_1 \cos 60^\circ - F_2 \cos 60^\circ - F_3$$

$$= 11.55 \cos 60^\circ - 2.31 \cos 60^\circ - 4.62 = 0 \text{ KN}$$

$$+\uparrow \Sigma V = F_1 \sin 60^\circ + F_2 \sin 60^\circ$$

$$= 11.55 \sin 60^\circ + 2.31 \sin 60^\circ = 12 \text{ KN}$$

Ans.

Ex. 1.27. Find magnitude of resultant force of a coplanar system as shown in Fig. Ex. 1.27. ABCDEF is a regular hexagon. Where will the resultant force intersect X and Y axis ? Find both distances.

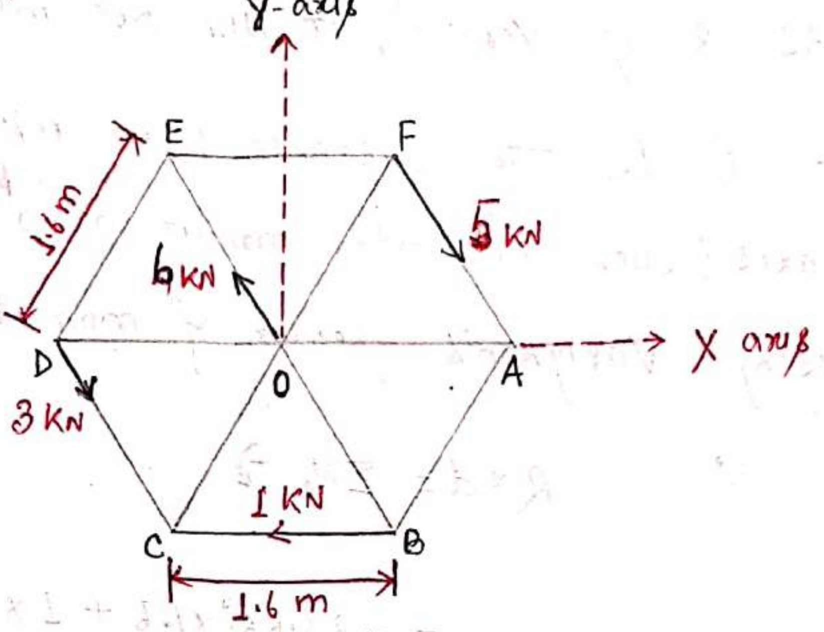


Fig: Ex 1.27

Soln : We know that all triangles in regular hexagon are equilateral.

Resolving all forces along X and Y axis.

$$\overset{+}{\rightarrow}\Sigma H = 3\cos 60^\circ + 5\cos 60^\circ - 6\cos 60^\circ - 1$$

$$\Sigma H = 0$$

Now, $+\uparrow\Sigma V = -3\sin 60^\circ - 5\sin 60^\circ + 6\sin 60^\circ$

$$\therefore \Sigma V = -1.732 \text{ KN} = 1.732 \text{ KN} (\downarrow)$$

$\therefore$ Magnitude of resultant and Direction;

$$\therefore R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$\therefore R = 1.732 \text{ KN} (\downarrow)$ ---- Ans

To find the position of resultant R

As 'R' is vertical, it will not intersect Y axis.

let 'd' be the distance where resultant R intersects x-axis (such that total moment at O produce clockwise).

using Varignon's theorem of moments

i.e $R \times d = \Sigma M_O$ (+↻)

$= -3 \sin 60^\circ \times 1.6 + 1 \times (0.8 \tan 60^\circ) + 5 \sin 60^\circ \times 1.6$

... other forces and components pass through O.

$= 4.156$ KN-m

$\therefore \quad d = \frac{4.156}{R} = \frac{4.156}{1.732}$

$d = 2.40$ m to the right of O ---- Ans

Ref. Fig. Ex 1.27(a).

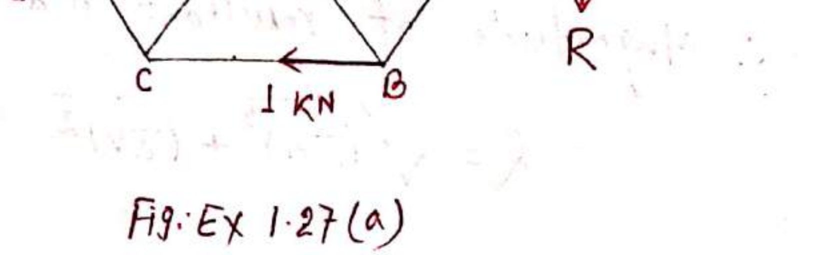


Fig: Ex 1.27 (a)

Problems based on parallel Forces

Following procedure is used to resolve the forces into parallel components.

When a force is required to resolve into two parallel components, we will come across following two cases.

1) Components lie on either sides of resultant
2) Components lie on one side of resultant.

Case 1: When two parallel components lie on either sides as shown in Fig---, we can find components using following concepts.

a) For this case both components must have same directions that of R.
b) If P and Q are components, $P+Q=R$
c) Use Varignon's theorem about the line of action of any one force to find magnitude of other force.

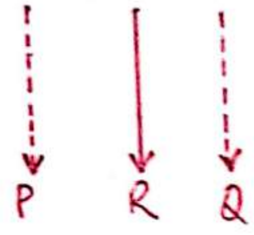


Fig:

Case 2: When two parallel components lie on one side of resultant as shown in Fig---, we can find components using following concepts.

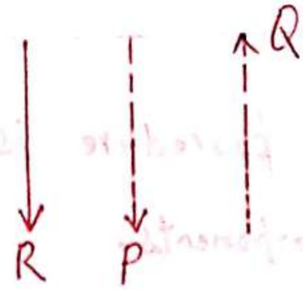


Fig :

a) For this Case the Components must have opposite directions.

b) The Component nearer to R would be in the same direction of that of R.

c) If P and Q are components, we have $P - Q = R$

d) Use Varignon's theorem about the line of action of any one force to find the magnitude of other force.

Ex 1·34 : Resolve the 600 N force at A into two parallel components P and Q acting respectively along

1) a − a and b − b

2) b − b and c − c

Also resolve the same force into a force P at B and a couple represent the couple by forces F acting along b-b and c-c.

Refer. Fig. Ex 1·34.

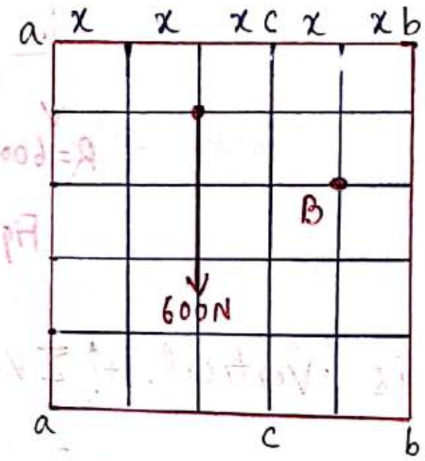


Fig: Ex 1·34

Soln:

Case 1 : Resolution into two parallel components acting along a-a and b-b (Refer. Fig. Ex. 1·34(a)).

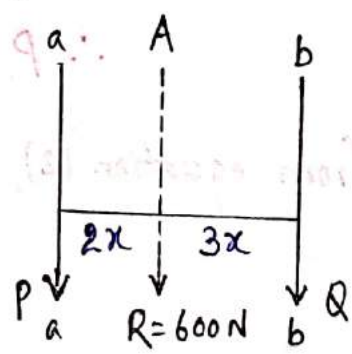


Fig: Ex 1·34(a)

As R is vertical

$$\therefore +\uparrow \Sigma V = R$$

$$\therefore -P - Q = -R$$

$$\therefore P + Q = 600\ N \quad \text{--- (1)}$$

Taking moment at b-b and using Varignon's theorem;

$600 \times 3x = P \times 5x$

$\therefore P = 360\,N\,(\downarrow)$ ... Ans

from equation (i), $Q = 240\,N\,(\downarrow)$ --- Ans.

Case 2: Resolution into two parallel components acting along b-b and c-c (Refer. Fig Ex 1.34 (b)).

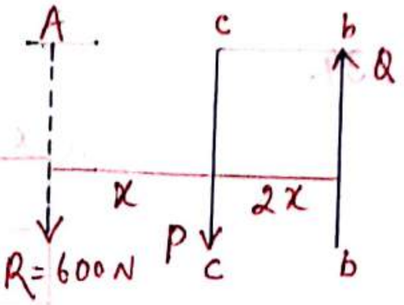


Fig. Ex 1.34 (b)

As R is vertical. $+\uparrow \Sigma V = R$

$\therefore -P + Q = -R$

$\therefore -P + Q = -600$ ————(2)

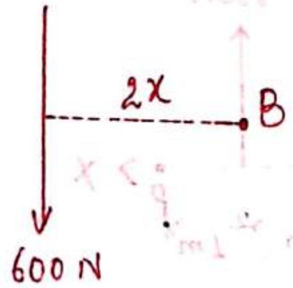
Taking moment at b-b and using Varignon's theorem.

$600 \times 3x = P \times 2x$

$\therefore P = 900\,N\,(\downarrow)$ ----- Ans

from equation (2), $Q = 300\,N\,(\uparrow)$ --- Ans.

Case 3 : Resolution into a single force and couple at B by application of principle of superposition (parallel transfer of a force)



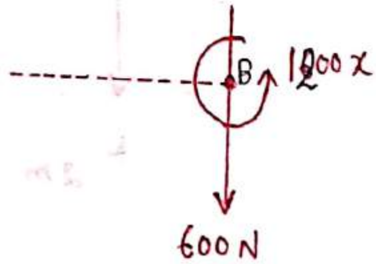


Fig: Ex. 1.34 (c) : Given System   Fig: Ex 1.34(d): Force couple system at B.

Moment about B , $M_B = 600 \times 2x = 1200\, x$

The moment at B = Moment of couple produced by two forces F acting at c-c and b-b.

$$1200\, x = F \times 2x$$

$\therefore F = 600$ N (downward at c-c, upward at b-b).

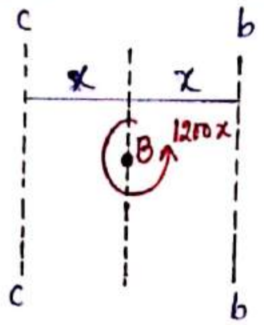


Fig. Ex 1.34 (e)

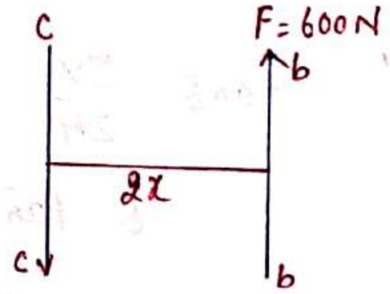


Fig: Ex 1.34 (f).

Ex 1.35 : Replace the force system by a single force resultant and specify its point of application measured along X-axis from point P. Refer Fig. Ex 1.35.

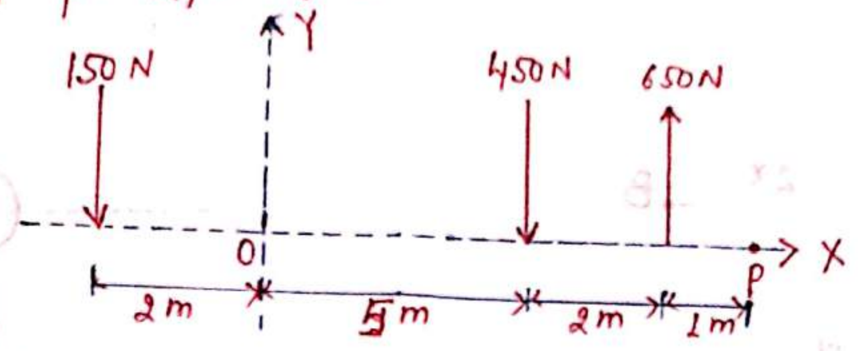


Fig. Ex 1.35

Soln: To find resultant

$\Sigma H = 0$ (No horizontal force)

$+\uparrow \Sigma V = -150 - 450 + 650$

$\therefore \Sigma V = 50 \text{ N} (\uparrow)$

$\therefore$ Magnitude of resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 50 \text{ N} \quad \text{--- Ans}$$

Direction;

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\therefore \theta = \tan^{-1}\left[\frac{50}{0}\right] = 90^\circ$$

$\therefore$ R is vertical force acting upward. -- Ans.

To find point of application:

Using Varignon's theorem about P

$\therefore \quad R \times d = \Sigma M_P \; (+\circlearrowleft)$

$$= 650 \times 1 - 450 \times 3 - 150 \times 10$$

$$= -2200 \text{ N-m} = 2200 \text{ N-m} \circlearrowright$$

$$\therefore \; d = \frac{2200}{50} = 44 \text{ m from } P$$

As total moment at P is anticlockwise, resultant must be acting to the right of P. Refer Fig. Ex 1.35(a)

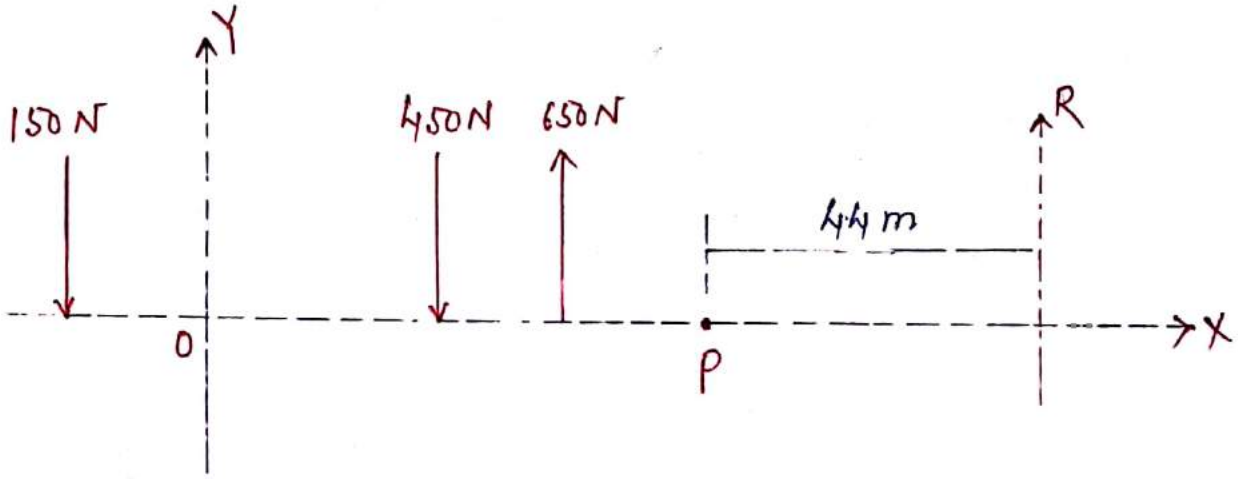
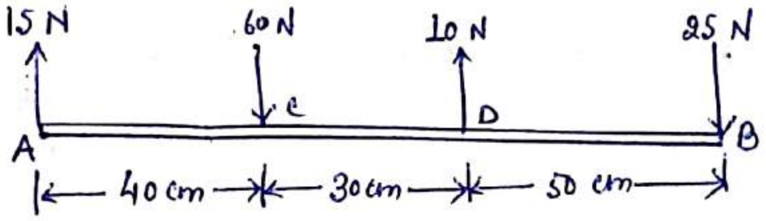


Fig. Ex. 1.35(a)

(Prob). A rigid bar is subjected to a system of parallel forces as shown in fig.



Reduce this system to

(a) single force

(b) a single force-moment system at A

(c) a single force-moment system at B.

Solu^n: (a) A single force (or resultant).

($\uparrow$+ve) let the resultant be R_y.

$$\therefore \quad R_y + 15 - 60 + 10 - 25 = 0$$

$$\therefore R_y = 60 \text{ N}$$

(but this will act in downward dir^n)

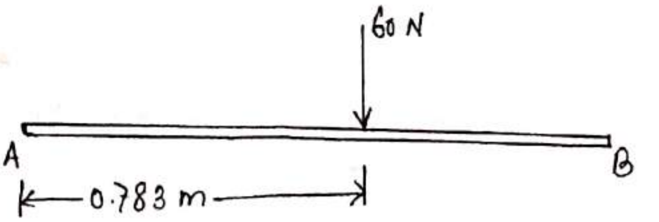
let the resultant R_y act at a distance x from A

M @ A;

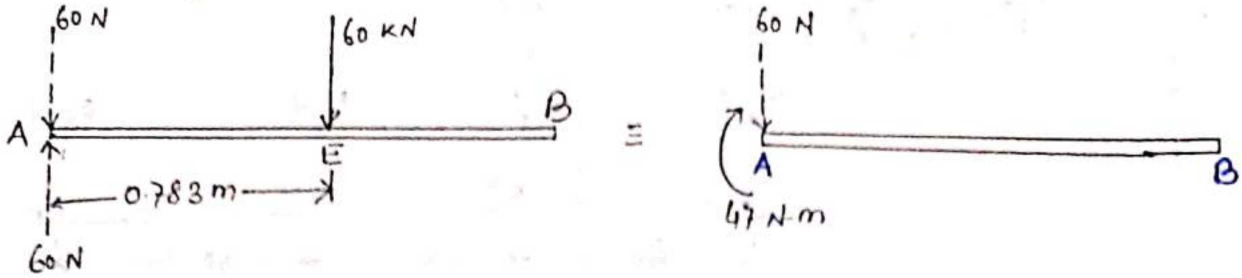
+↻ $$x \times R_y \cancel{+} 60 \times 0.4 + 25 \times 1.2 - 10 \times .7 = 0$$

$$\therefore x \cancel{\times} (60) \cancel{+} 24 + 30 - 7 = 0$$

$$x = +0.783 \text{ m from A.}$$



(b) Single force-moment at A;

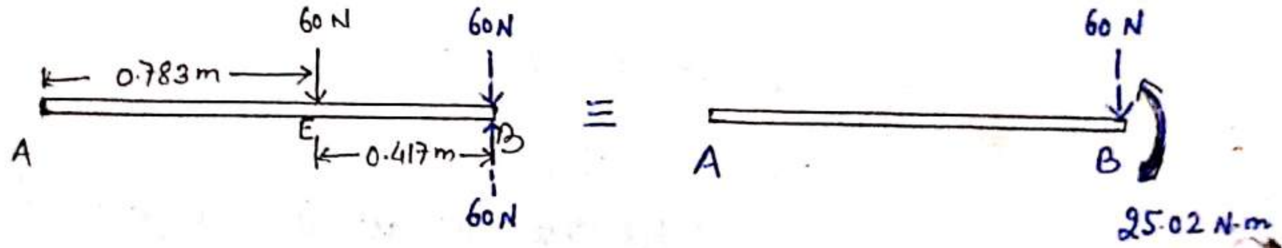


If the force of 60 N acting at E is moved to the point A. It is accompanied by a moment

$$M_A = 60 \times 0.783$$

$$M_A = 47 \text{ N-m}$$

(c) Single force-moment at B:



If the force of 60 N acting at E is moved to the point B, it is accompanied by a moment

$$M_B = 60 \times 0.417$$

$$M_B = 25.02 \text{ N-m}$$

Ans.

Problems based on Moment of a force and Couple

Ex. 1.36: points A, B and C lying in x-y plane. The moment of a certain force 'F' acting in x-y plane is 180 N-m clockwise about the origin O, 90 N-m anticlockwise about point B. of its moment @ point A is zero. Determine the magnitude and direction of force 'F' and the moment of force 'F' @ point C.

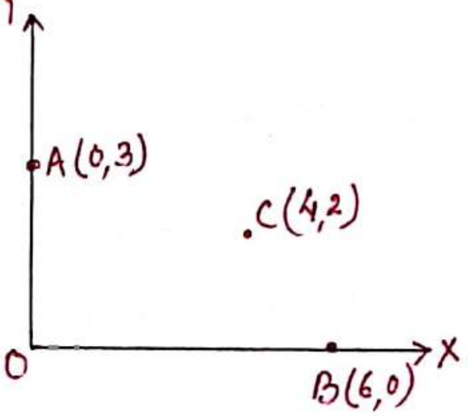


Fig. Ex. 1.36

Soln: To find moment of force @ point C, first find magnitude and direction of force in x-y plane.

Given moments are

$$M_O = +180 \text{ N-m}$$

$$M_B = -90 \text{ N-m}$$

$$M_A = 0$$

As moment of force F @ point A is zero, hence force must pass through point A.

Assume force F acting in 4th quadrant.
(You can assume the force in any quadrant).

Now, using

$$M_O = 180 = F\cos\theta \times 3$$

$$\therefore F\cos\theta = 60 \quad --- (i)$$

and $M_B = -90 = -F\sin\theta \times 6 + F\cos\theta \times 3$ (Refer Fig. Ex 1.36 (a)).

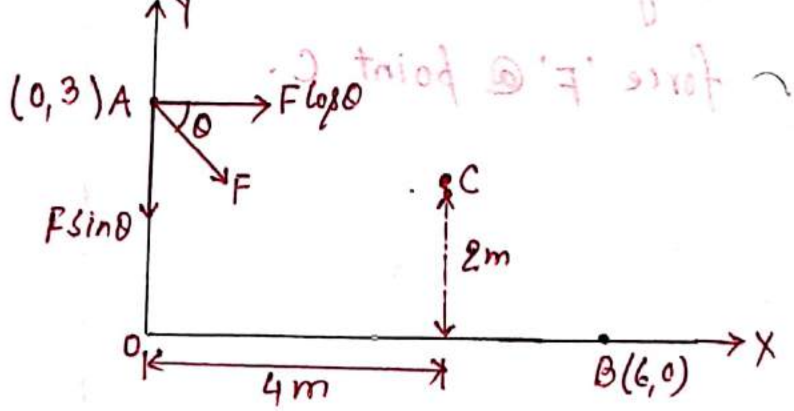


Fig. Ex 1.36 (a)

$$\therefore -90 = -F\sin\theta \times 6 + 60 \times 3 \quad [\because \text{from equation (i)}]$$

$$\therefore F\sin\theta = 45 \quad --- (ii)$$

Now, from equation (i) and (ii), we get

$$\frac{F\sin\theta}{F\cos\theta} = \frac{45}{60}$$

$$\therefore \tan\theta = 0.75 \qquad \therefore \theta = 36.87^\circ \quad --- \text{Ans.}$$

Substituting this value in equation (i), We get, $F = \frac{60}{\cos 36.87^\circ}$ $\therefore F = 75$ N --- Ans.

Now, moment of force @ point C,

$$M_c = -F\sin\theta \times 4 + F\cos\theta \times 1$$

$$= -75 \times \sin 36.87^\circ \times 4 + 75 \times \cos 36.87^\circ \times 1$$

$$= -120 \text{ N-m} = 120 \text{ N-m (anticlockwise)}$$ -- Ans.

Ex 1.37 : The moments of a given plane system of forces about three points A (0,1), B(2,0) and C (2,2) are $M_A = +36$, $M_B = +3$ and $M_c = +21$ unit respectively. Find the magnitude and direction of the resultant force of the force system.

Soln: The points A(0,1), B(2,0) and C(2,2) are shown in Fig. Ex 1.37.

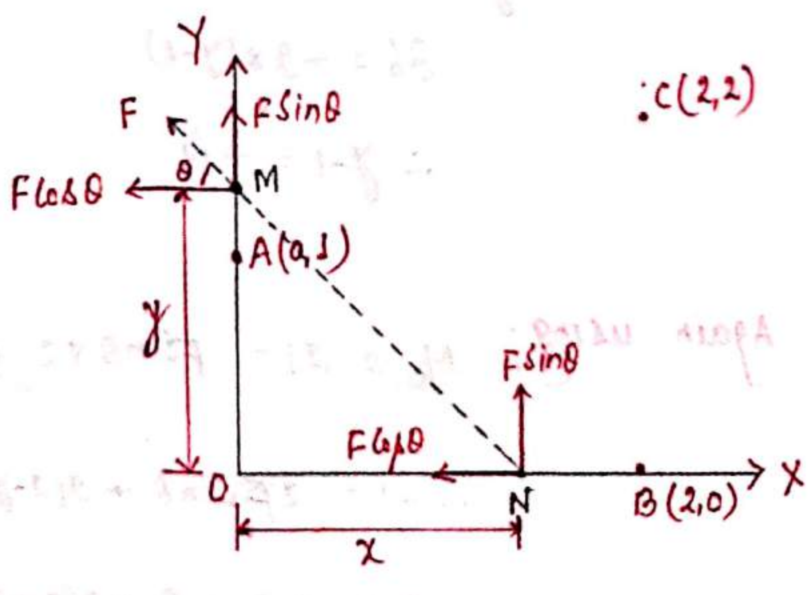


Fig. Ex 1.37.

Given moments are

$$M_A = +36 \text{ units}$$
$$M_B = +3 \text{ units}$$
$$M_C = +21 \text{ units.}$$

Now, assume that force F is acting in 2nd quadrant.

Let Co-ordinates of intersection points be

M(0, y) and N(x, 0) as shown in Fig. Ex 1.37

Now, using

$$M_A = 36 = -F\cos\theta(y-1) \quad \text{---(i)}$$
$$M_B = 3 = F\sin\theta(2-x) \quad \text{---(ii)}$$
$$M_C = 21 = F\sin\theta(2-x) + F\cos\theta \times 2 \quad \text{---(iii)} \quad \text{...(resolving at N)}$$

Substituting $F\sin\theta(2-x) = 3$ in equation (iii), we get

$$21 = 3 + 2F\cos\theta$$
$$\therefore F\cos\theta = 9 \ (\leftarrow) \qquad \therefore \cancel{F\cos\theta = 9}$$

Substituting this value in equation (i)

$$36 = -9 \times (y-1)$$
$$\therefore y-1 = -4 \qquad \therefore y = -3 \text{ units}$$

Again using; $M_C = 21 = F\sin\theta \times 2 + F\cos\theta(2-y)$ ...(resolving at M)

$$\therefore 21 = 2F\sin\theta + 9(2-y)$$
$$21 = 2F\sin\theta + 9(2+3)$$

$\therefore \; F \sin\theta = -12$ $\qquad\qquad \therefore F\sin\theta = 12 \downarrow$

Substituting this value in equation (ii), we get

$$3 = -12(2-x)$$

$$2-x = -0.25 \qquad\qquad \therefore x = 2.25 \text{ unit.}$$

Magnitude and direction of force.

$$\frac{F\sin\theta}{F\cos\theta} = \frac{12}{9} \qquad\qquad \therefore \theta = 53.13^\circ$$

$$\therefore \; F = \frac{12}{\sin 53.13^\circ} \qquad\qquad \therefore F = 15 \text{ units.}$$

$\therefore$ Force of 15 units acting in IIIrd quadrant making angle of 53.13° with horizontal intersects at $(2.25, -3)$ units

Refer. Fig. EX 1.37 (a).

Check:

$M_A = 9 \times 4 = +36$ units

$M_B = 9\times3 - 12\times2 = +3$ units

$M_C = 9\times5 - 12\times2 = +21$ units.

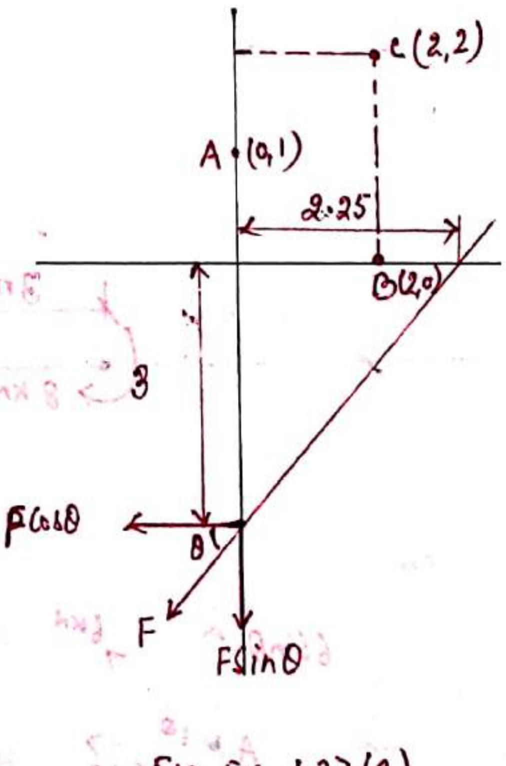


Fig. EX. 1.37 (a)

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Ex. 1·38 : Replace the force and the couple system in the (Fig. Ex 1·38) by an equivalent single force and single moment at P.

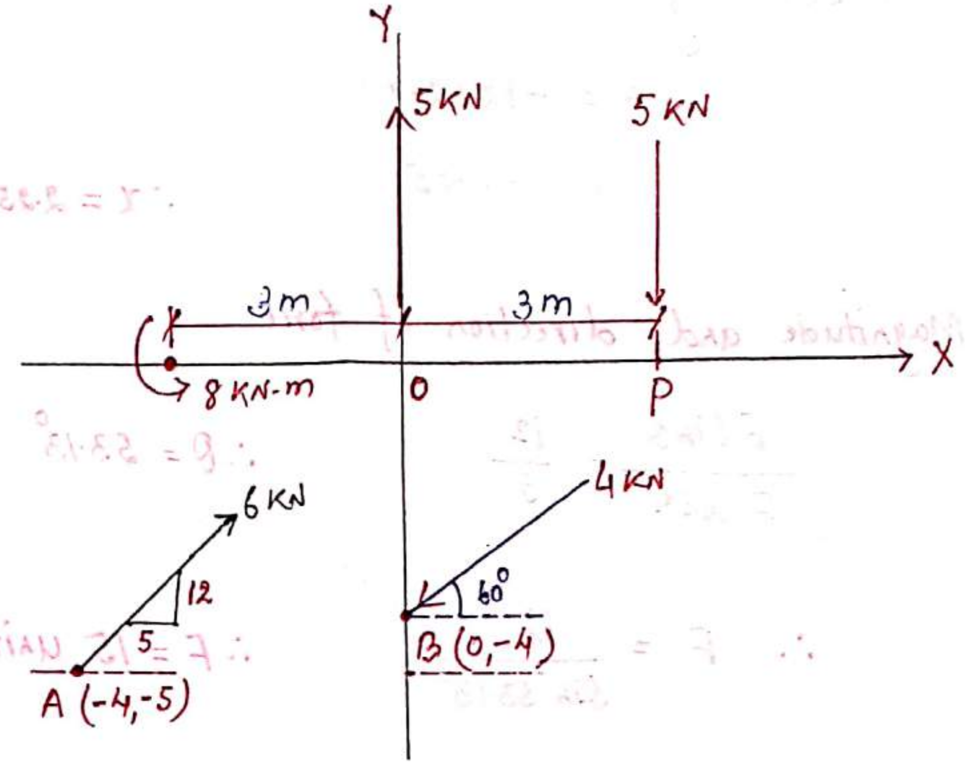

Fig. Ex 1·38

Soln: Redraw the figure for better understanding (Refer. Fig. Ex 1·38(a))

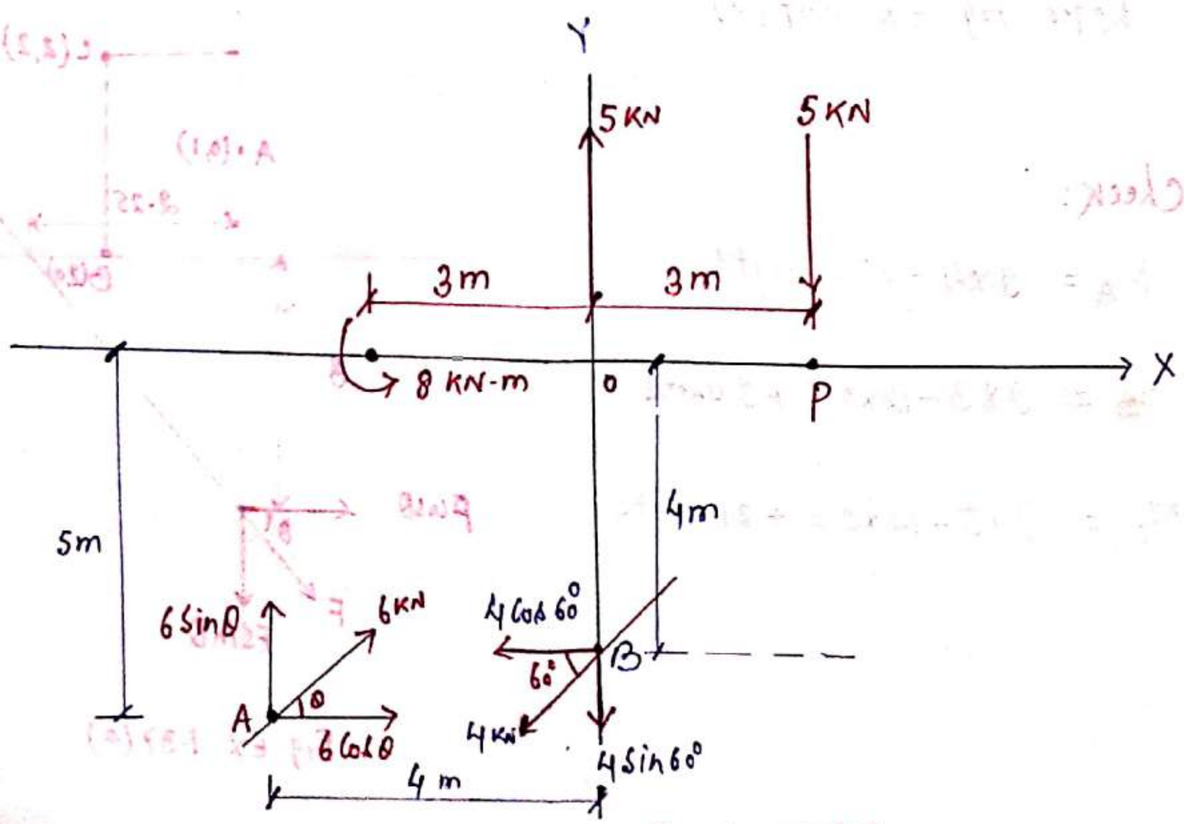

Fig. Ex 1·38(a)

To find a single force-system couple, first find equivalent single force and moment @ point P.

Equivalent single force (i.e Resultant force)

Resolving all forces along X and Y axis;

$\xrightarrow{+}\Sigma H = 6\cos\theta - 4\cos 60^\circ$

Now, $\tan\theta = \frac{12}{5}$ $\therefore \theta = 67.38^\circ$

$\therefore \Sigma H = 0.307$ KN $(\rightarrow)$

$\therefore +\uparrow \Sigma V = 6\sin\theta - 4\sin 60^\circ + 5 - 5$

$\therefore \Sigma V = 2.07$ KN $(\uparrow)$

Magnitude of resultant

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\therefore R = \sqrt{(0.307)^2 + (2.07)^2} = 2.093 \text{ KN} \quad \text{--- Ans.}$$

Direction;

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\therefore \theta = \tan^{-1}\left[\frac{2.07}{0.307}\right] = 81.56^\circ \quad \text{--- Ans}$$

Total moment at P : i.e ΣM_P +↺

$$\Sigma M_P = -6\cos\theta \times 5 + 6\sin\theta(4+3) + 4\cos 60^\circ \times 4 - 4\sin 60^\circ \times 3 + 5 \times 3 - 8$$

$\therefore \Sigma M_P = -11.54 + 38.76 + 8 - 10.4 + 15.8$

$\therefore M_P = 31.82$ KN-m (i.e clockwise). --- Ans.

Refer. Fig. EX 1.38 (b).

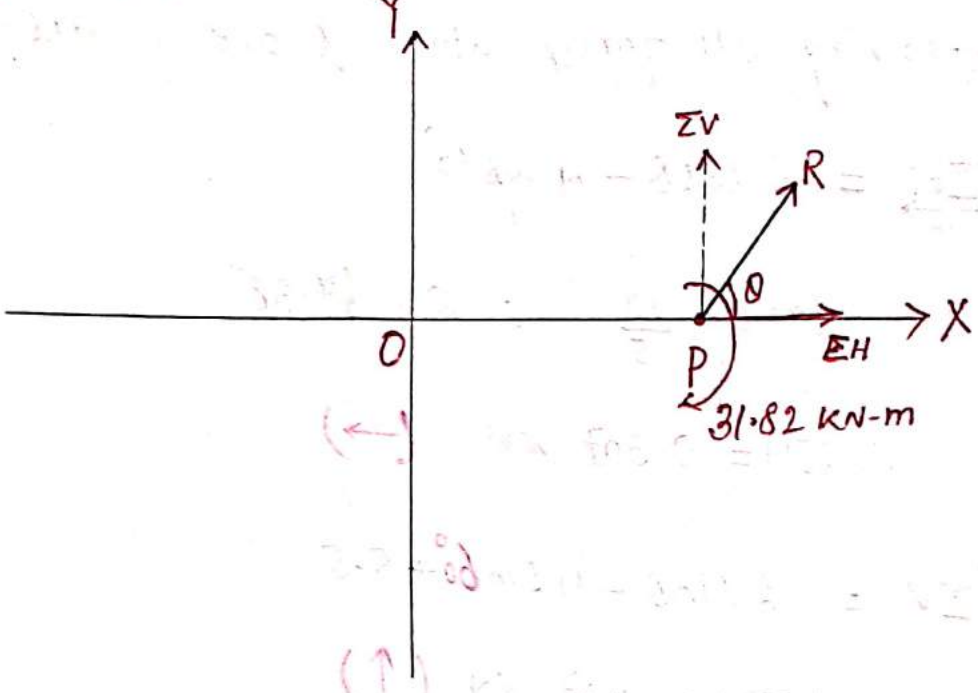

Fig: EX 1.38 (b) : Equivalent force-couple system at P.

CHAPTER 2

Equilibrium & Analysis of Beams

Part A – Equilibrium

part B – Analysis of Beams.

Introduction :

In this chapter we shall derive the relationship between various forces acting on a particle / body in a state of equilibrium. We shall also learn free body diagram which are the key factors in the analysis of forces in equilibrium.

2.1. Equilibrium:

When the condition of the body is unaffected even though a number of forces acted upon it, it is said to be in equilibrium.

Hence, equilibrium requires that a particle or body either be at rest, if originally at rest or move with a constant velocity, if originally moving with a velocity.

2.2. Analytical conditions of Equilibrium:

There are the following conditions of Equilibrium for ~~concurre~~ coplaner forces.

a) Co-planer Concurrent Forces:

For coplaner concurrent forces, there are two conditions of equilibrium.

i.e $\Sigma F_H = 0$ and $\Sigma F_V = 0$ $\therefore R = 0$

b) Coplaner non-concurrent forces:

For coplaner non-concurrent forces, there are three conditions of equilibrium.

$\therefore \Sigma F_H = 0, \Sigma F_V = 0$ and $\Sigma M = 0$

2.3. Equilibrium under Different Forces:

2.3.1. Equilibrium under Single force:

Equilibrium under single force does not exist.

2.3.2. Equilibrium under two forces:

When a body is in Equilibrium under only two forces, they must be equal, opposite and Collinear. (Two force system).

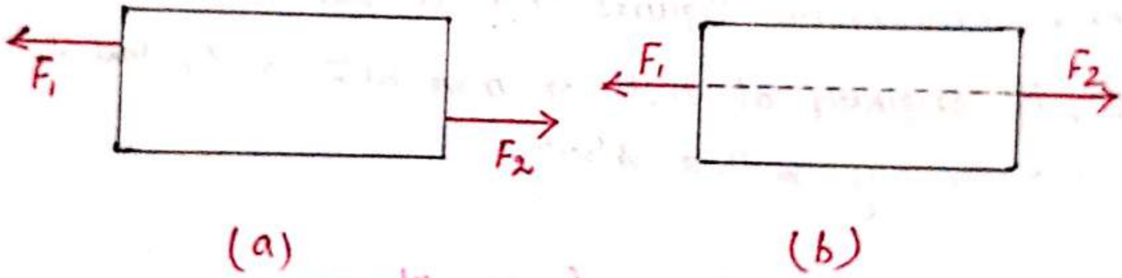

Fig 2.

System shown in Fig 2(a) subjected to two forces F_1 and F_2, equilibrium is not possible even if $F_1 = F_2$.
For equilibrium they must be equal, opposite and Collinear as shown in Fig 2(b).

2.3.3. Equilibrium under 'Three' forces:

When a body is in Equilibrium under three forces, then the resultant of two forces must be equal, opposite and Collinear with the third force (Three force system).

2.3.3.1. Special condition of Equilibrium under Three forces:

Lami's theorem:

It States " If Three Coplanar Concurrent forces acting on a body are in equilibrium, then each force is proportional to the Sine of the angle between the other two forces."

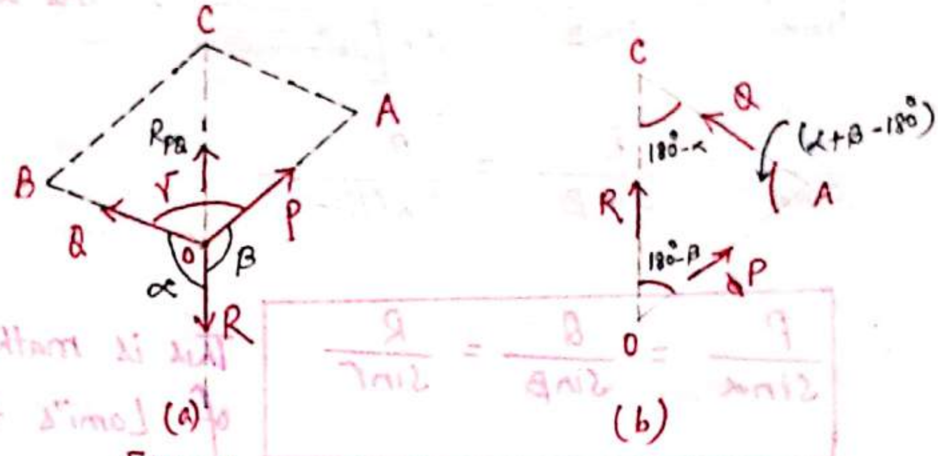

Fig: 3

Let P, Q and R be the three forces acting at point O in equilibrium.

Draw a parallelogram OACB with OA = P and OB = Q.

As per the law of parallelogram of forces, diagonal OC will be the resultant R_{PQ} of the two forces P and Q.

Now, point O is subjected to only two forces R and R_{PQ}.

As per equilibrium under two forces, R and R_{PQ} must be equal, opposite and Collinear.

Now, by Sine rule in Δ OAC,

$$\frac{P}{\sin \angle ACO} = \frac{Q}{\sin \angle COA} = \frac{R_{PQ} = R}{\sin \angle OAC}$$

$$\therefore \frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180^\circ-\beta)} = \frac{R}{\sin[\alpha+\beta-180^\circ]}$$

$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin[360^\circ-\gamma-180^\circ]} \quad \ldots \left[\text{As } \alpha+\beta+\gamma = 360^\circ\right]$$

$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin(180-\gamma)}$$

$$\therefore \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

This is mathematical form of Lami's theorem.

2.3.3.2. Limitations:

There are the following limitations of Lami's theorem.

1) This is applicable for only three forces.
2) Forces should be concurrent force system.
3) Nature of forces must be same.

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2.3.4. Equilibrium under "FOUR OR MORE" Forces:

When a body is in equilibrium under four or more than four forces, apply conditions of equilibrium.

If forces are Co-planer Concurrent, use $\Sigma F_H = 0$ and $\Sigma F_V = 0$

If forces are Coplaner Non-concurrent, use $\Sigma F_H = 0$, $\Sigma F_V = 0$ and $\Sigma M = 0$.

2.4. Constraint, Action and Reaction:

The restriction to the motion of a body in any direction is called a Constraint.

e.g. When a ball is resting on smooth surface, horizontal motion is possible but vertical downward motion is restricted by plane. (Refer Fig. 4)

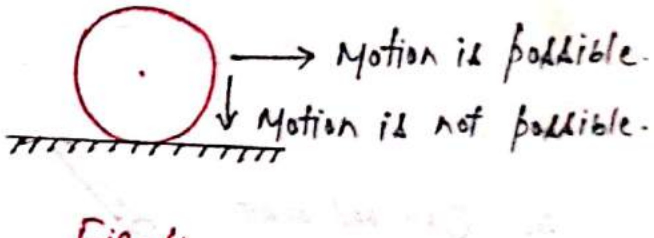
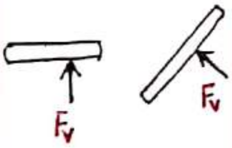
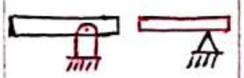
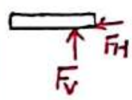
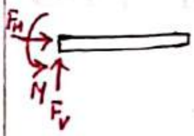
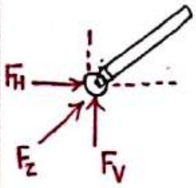
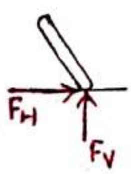


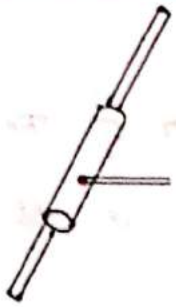
Fig: 4

In general, the action of a constrained body on any support induces an equal and opposite reaction from the support.

2.5. Types of Supports and Corresponding Reactions :

The table given below will provide an idea to identify the reactions for different types of supports.

Sr. No.	Support/Connection	Sketch	Reaction	Specification	No. of unknowns
1.	Rollers			Reaction which is perpendicular to plane on which roller rests.	One.
2.	Smooth pin or Hinge.			Two reaction components with unknown directions.	Two
3.	Fixed Support			Two reaction components and one moment with all components unknown in directions	Three.
4.	Ball and Socket Joint			Three reaction components in unknown directions.	Three
5.	Flexible Cord or Cable.			one axial force acting away from body (Tension)	One
6.	Smooth Surface			Reaction is $\perp^r$ to the surface.	One
7.	Rough Surface			Two reaction components with unknown directions	Two.

Sr. No.	Support/Reaction	Sketch	Reaction	Specification	No. of unknown
8.	A sliding Collar.			Reaction is $\perp^r$ to the rod along which Collar is sliding without friction.	One.

2.6. Free Body Diagram (F.B.D):

An isolated body separated from all other connected bodies or surfaces is called free body.

A free body diagram is a diagram or sketch of an isolated body or system of bodies showing

a) all active forces, such as applied forces and gravity forces; and

b) all reactive forces; the reactive forces are supplied by wall, pins, rollers, cables or other means.

The free body diagram is the most important tools in the analysis of Mechanics's problems.

2.7. Procedure for the solution of problems in Equilibrium

There are the following steps for the solution of problems in Equilibrium :

1) Determine what data are given and what results are required.

2) Draw a free body diagram of the member or group of members on which some or all of the unknown forces are acting.

3) Observe the type of force system, which acts on the free body diagram.

4) Note the number of independent equations of equilibrium available for the type of force system involved.

5) Compare the number of unknowns on the free body diagram with the number of independent equations available for the force system.

6) 1. If there are as many independent equations as unknowns, proceed with the solution by writting and solving the equations.

2. If there are more unknowns to be evaluated than independent equations,
draw a F.B.D. of another body and repeat steps 3, 4 and 5 for the second free body diagram.
proceed with the solution by writting and solving the equations.

2.8. Use of Free Body Diagram in Statics:

There are the following uses of F.B.D. in the problems of statics:

1). The problems involving equilibrium of bodies under any system of forces can be simplified by drawing free body diagram of each body separately.

2). All equations of equilibrium can be applied to each free body diagram.

3). The unknown forces for equilibrium of each body can be obtained very easily.

Problems based on Free Body Diagram.

Ex. 2.A.1: Draw the F.B.D. of a sphere of weight 'W' resting on a frictionless plane surface. [Refer Fig. Ex. 2.A.1]

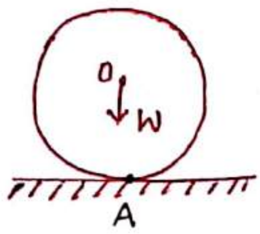


Fig: Ex. 2.A.1

Soln: The free body diagram is shown in Fig. Ex. 2.A.1(a).

i.e. The sphere is in equilibrium under the action of two equal and opposite forces. W and R_A, which are collinear.

Fig. Ex. 2.A.1(a)

Ex. 2.A.2: Draw the F.B.D of a sphere of weight 'W'. [Refer. Fig. Ex. 2.A.2]

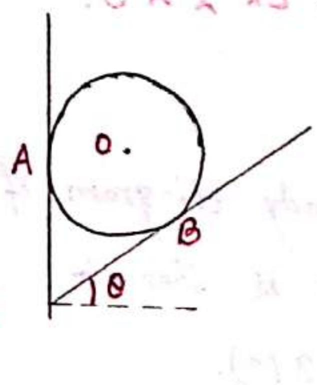


Fig. Ex. 2.A.2.

Soln: The free body diagram is shown in Fig. Ex. 2.A.2(a).

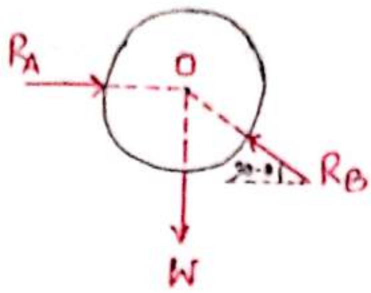


Fig. Ex. 2.A.2(a).

Ex. 2.A.3: Two spheres A and B, each of weight W_A and W_B respectively [Refer Fig Ex. 2.A.3] resting on an inclined plane. Draw F.B.D. of each sphere.

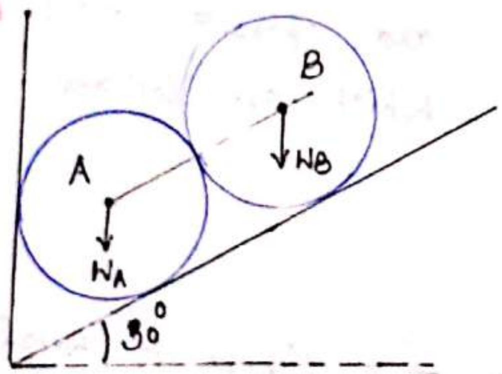
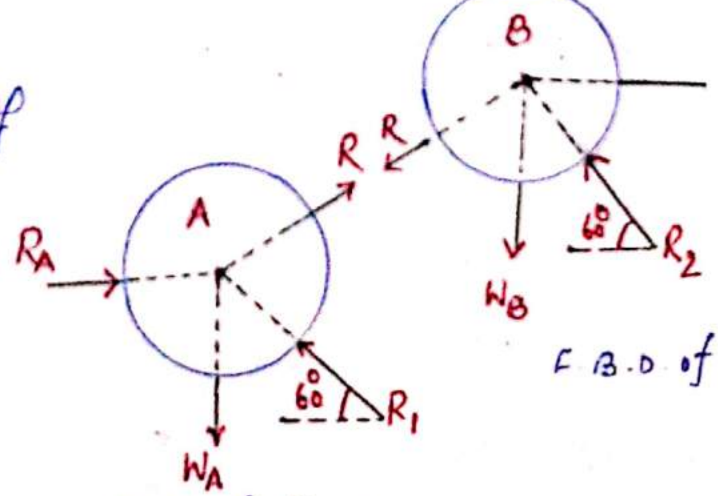


Fig: Ex. 2.A.3.

Soln:

The free body diagram of each sphere is shown in Fig. Ex. 2.A.3(a).



F.B.D of sphere B

F.B.D of sphere A

Fig. Ex. 2.A.3(a)

Ex. 2.A.4 : Two similar spheres P and Q each of weight 'W' rest inside a hollow cylinder which is resting on a horizontal plane. Draw the F.B.D of

1) both the spheres taken together.

2) the sphere P

3) the sphere Q. Refer. Fig. Ex. 2.A.4.

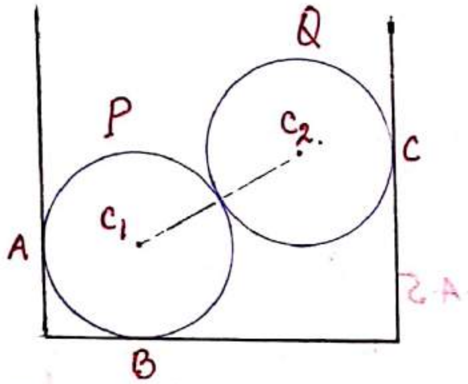


Fig. Ex. 2.A.4.

Soln : The following are the F.B.D. of spheres. Refer Fig. 2.A.4 (a)

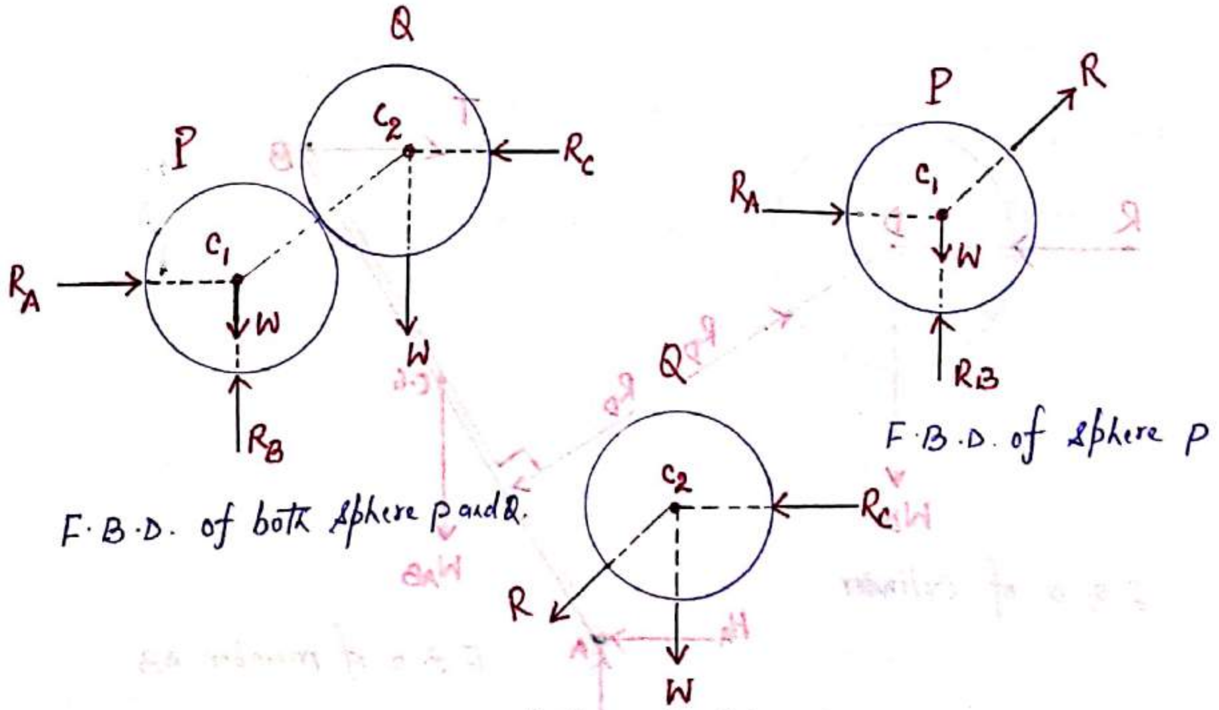


Fig. Ex. 2.A.4 (a).

Ex. 2.A.5 : Draw F.B.D. for the member AB and cylinder D. Neglect friction at the contact surface of the cylinder. The weight of cylinder and the member are denoted as W_D and W_{AB} respectively. (Refer. Fig. Ex. 2.A.5).

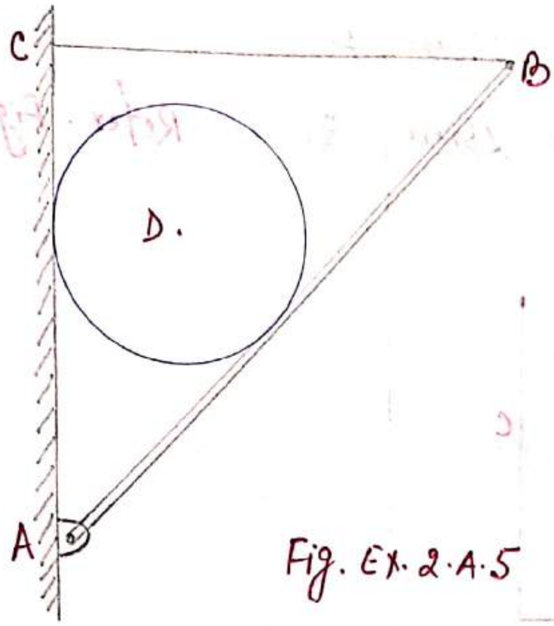


Fig. Ex. 2.A.5

Soln : The free body diagram of cylinder and member AB is shown in Fig. Ex. 2.A.5(a).

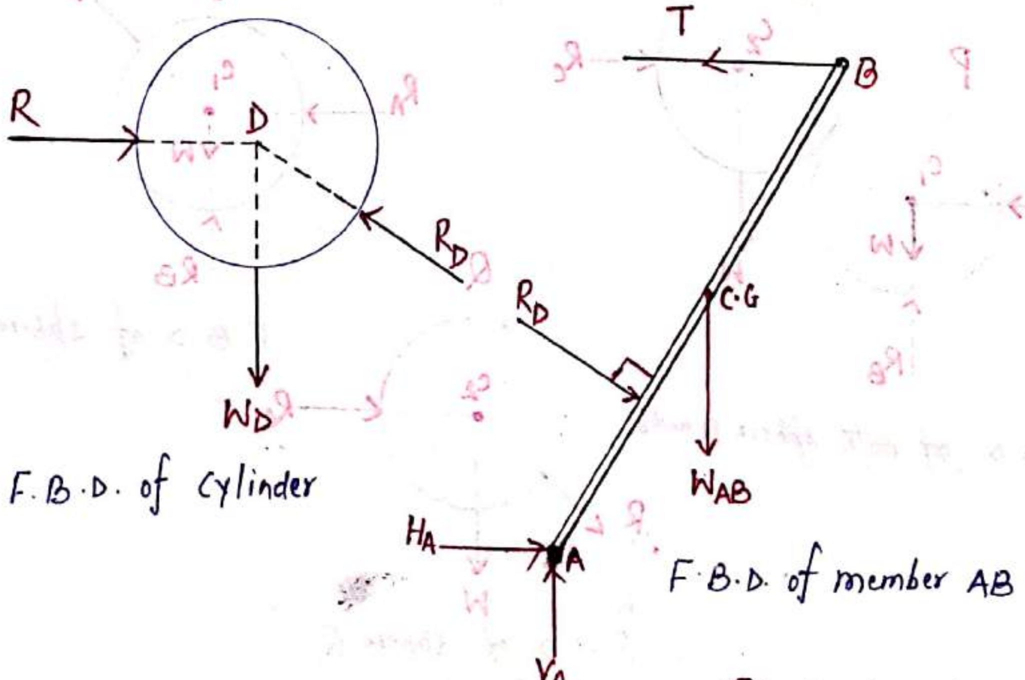


Fig. Ex. 2.A.5(a)

Problems based on Equilibrium of circular Bodies

Ex. 2.A.9: Two smooth spheres, each of radius r and weight Q, rest in a horizontal channel having vertical walls, the distance between which is b. Find the pressures exerted on the walls and floor at the points of contact A, B and D. The following data are given

$$r = 250 \text{ mm}; \quad b = 900 \text{ mm}, \quad Q = 500 \text{ N}.$$

Refer. Fig. Ex 2.A.9.

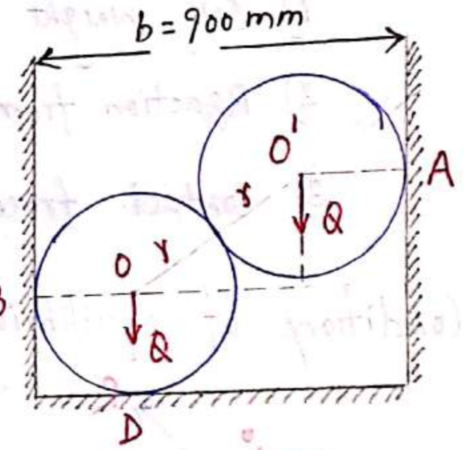


Fig. Ex. 2.A.9

Soln: Draw F.B.D. of spheres.

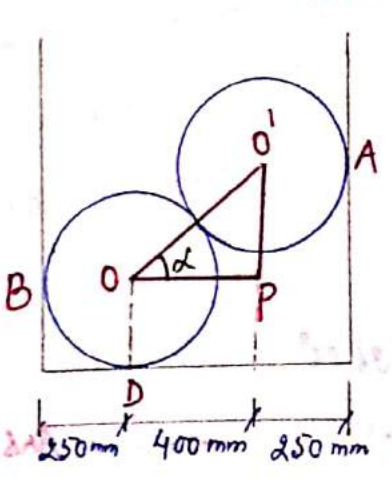


Fig. Ex. 2.A.9 (a)

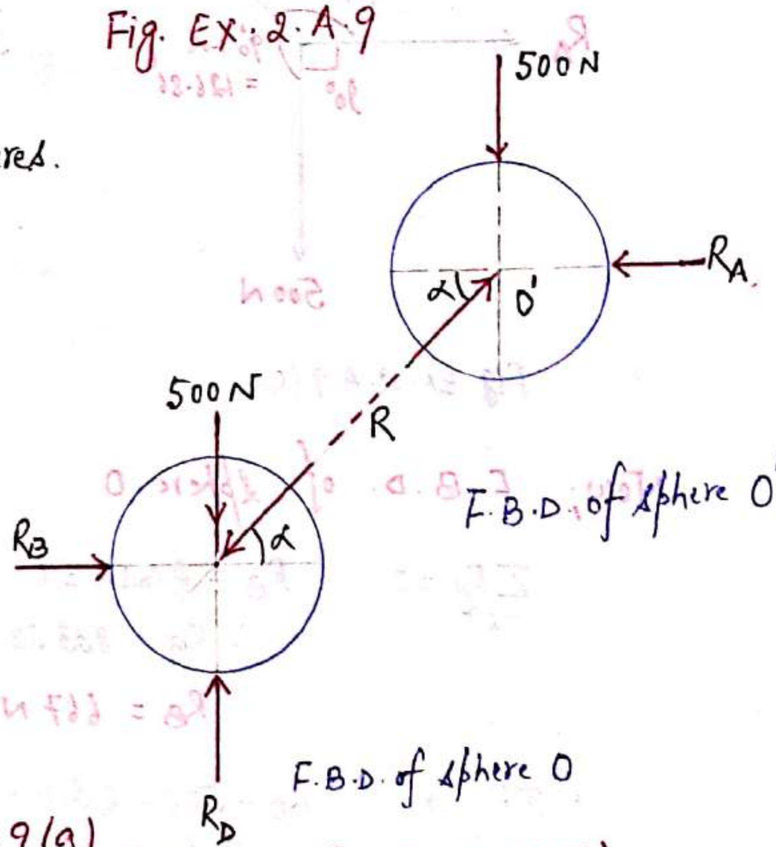


Fig. Ex. 2.A.9 (b)

from geometry : Refer. Fig. Ex. 2 A 9 (a)

$$OO' = r_1 + r_2 = 250 + 250 = 500 \text{ mm}$$

$$OP = 900 - 250 - 250 = 400 \text{ mm}$$

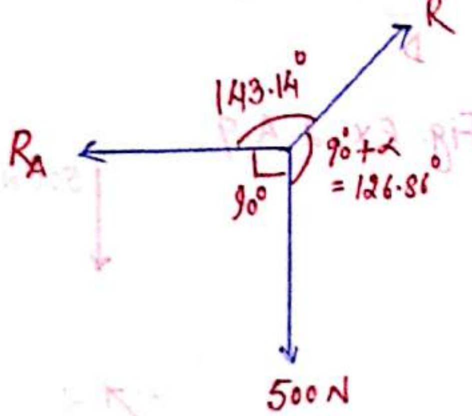
In $\Delta OPO'$, $\cos\alpha = \dfrac{OP}{OO'} = \dfrac{400}{500}$

$$\therefore \alpha = 36.86^\circ$$

F.B.D of ~~roller~~ sphere O' involved;

1) Self weight 500 N
2) Reaction from vertical wall at A i.e R_A
3) Contact froce between sphere i.e R.

∴ Conditions of equilibrium : Apply' Lami's theorem at O'



$$\frac{R_A}{\sin 126.86^\circ} = \frac{500}{\sin 143.14^\circ} = \frac{R}{\sin 90^\circ}$$

$$R = 833.52 \text{ N}$$

$$\therefore R_A = 667 \text{ N} (\leftarrow) \quad \text{---- Ans.}$$

Fig. Ex. 2 A.9 (c)

Now; F.B.D. of sphere O

$\overset{+}{\rightarrow} \Sigma F_H = 0 \qquad R_B - R\cos\alpha = 0$

$$\therefore R_B = 833.52 \cos 36.86^\circ$$

$$R_B = 667 \text{ N} (\rightarrow) \quad \text{---- Ans}$$

$+\uparrow \Sigma F_y = 0 \qquad R_D - 500 - R\sin\alpha = 0$

$$\therefore R_D = 500 + 833.52 \sin 36.86^\circ$$

$$R_D = 1000 \text{ N} (\uparrow) \quad \text{--- Ans.}$$

Ex. 2.A.10. A roller of radius $r = 300$ mm and weight $W = 2500$ N is to be pulled over a Curb of height $h = 150$ mm by a horizontal force P applied to the end of a string wound around the circumference of the roller. Find the magnitude of 'p' required to start the roller over the curb. Refer. Fig. Ex. 2.A.10.

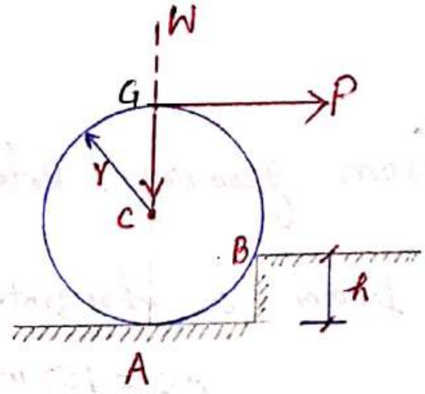


Fig: Ex. 2.A.10.

Soln: F.B.D. of roller, which involves:

1) Applied force P
2) Self weight $W = 2500$ N
3) Reaction from corner B of curb. i.e R_B

Note that reaction R_A reduces to zero when roller is just beginning to roll.

Since the roller is in equilibrium under three forces, so they must be concurrent at G. (P and weight W both are passing through G, hence reaction R_B must passing through G).

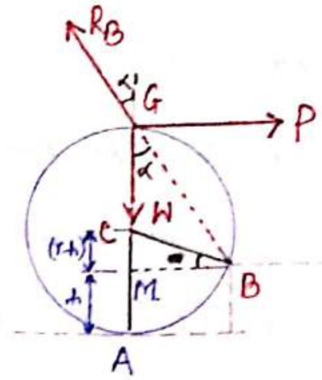


Fig. EX. 2. A. 10 (a)

from geometry : Refer Fig. Ex. 2. A. 10(a)

Draw a horizontal line BM from B and join BC.

$AC = 150$ mm

$\therefore CM = r - h = 300 - 150 = 150$ mm

$CB = r = 300$ mm.

$\therefore$ In Δ BMC,

$$BM = \sqrt{(BC)^2 - (CM)^2} = \sqrt{(300)^2 - (150)^2} = 259.80 \text{ mm}$$

Now, In Δ BMG,

$$\tan\alpha = \frac{BM}{GM} = \frac{259.80}{450}$$

$$\therefore \alpha = 30^\circ.$$

Now; Conditions of equilibrium: Apply Lami's theorem at G.

Refer. Fig. Ex. 2. A. 10(b)

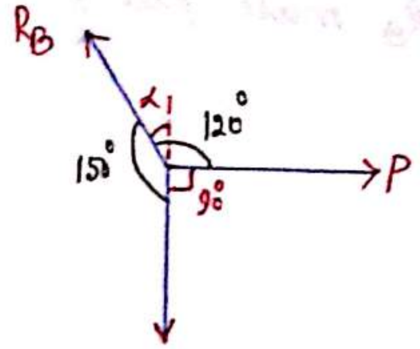


Fig. EX. 2. A. 10(b)

$$\therefore \frac{P}{\sin 150^\circ} = \frac{2500}{\sin 120^\circ}$$

$$\therefore P = 1443.4 \text{ N}$$ --- Ans.

Ex. 2.A.11: A uniform wheel 800 mm in diameter rests against a rigid rectangular block 180 mm thick. Find the pull through the centre of the wheel to just turn the wheel over the corner of the block if the weight of wheel is 600 N. Also find the reaction of the block. Refer. Fig. Ex. 2.A.11.

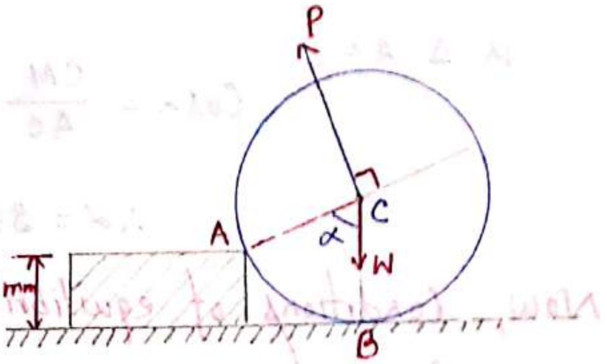


Fig. Ex. 2.A.11.

Soln: F.B.D. of wheel, which involves:

1) applied force P
2) Self weight $W = 600\ N$
3) Reaction from corner of rigid rectangular block i.e. R_A.

Since the wheel is in equilibrium under the three forces, so that they must be concurrent at C. (P and weight both are passing through C, hence reaction R_A must passing through C).

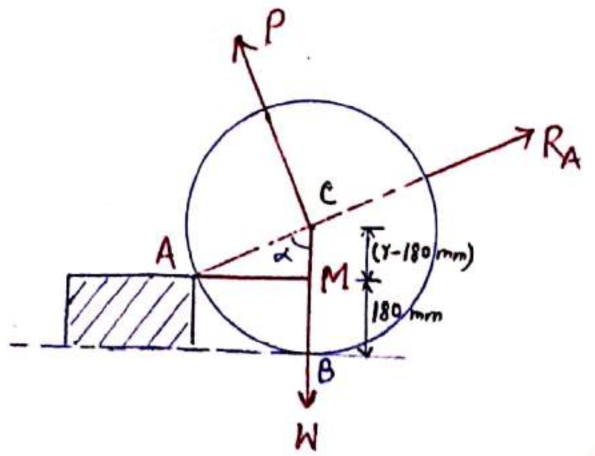


Fig. Ex. 2.A.11(a)

from geometery : Refer Fig. Ex. 2. A. 11(a)

Draw a horizontal line AM from A and Join AC.

$MB = 180\ mm$;

$\therefore CM = r - 180 = 400 - 180 = 220\ mm$

$AC = 400\ mm$.

In Δ AMC, $\cos\alpha = \dfrac{CM}{AC} = \dfrac{220}{400}$

$\therefore \alpha = 56.63^\circ$

Now, Conditions of equilibrium : Apply Lami's theorem at C.

Refer. Fig. Ex. 2. A. 11(b)

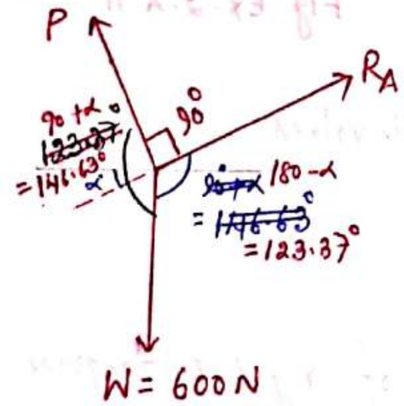


Fig. Ex. 2.A.11(b)

$$\therefore \frac{P}{\sin 123.37^\circ} = \frac{600}{\sin 90^\circ} = \frac{R_A}{\sin 146.63^\circ}$$

$$\cancel{\therefore P = \frac{600 \times \sin 146.63^\circ}{\sin 90^\circ}} =$$

$$\therefore P = \frac{600 \times \sin 123.37^\circ}{\sin 90^\circ} = 501.08\ N \quad \text{--- Ans.}$$

$$R_A = \frac{600 \times \sin 146.63^\circ}{\sin 90^\circ} = 330.02\ N \quad \text{--- Ans}$$

Ex. 2.A.12: A homogenous disc of weight W resting against an obstacle is to be just turned over the corner of obstacle by means of force P. Find the angle θ so that force P will be minimum. Also find P in terms of W. Assume $h = 0.25r$. Refer Fig. Ex. 2.A.12.

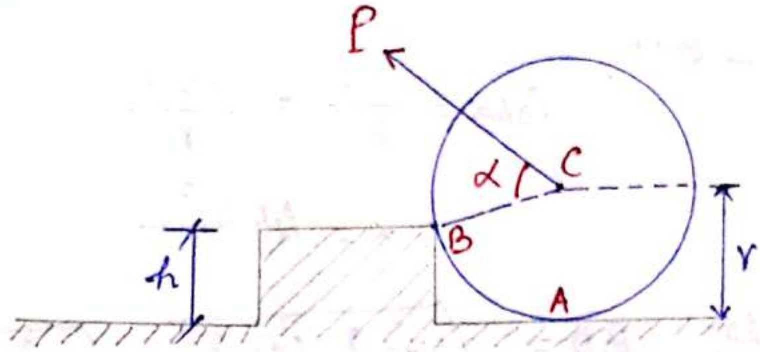


Fig. Ex. 2.A.12

Soln: F.B.D. of disc which involves:

1) applied force P
2) Self weight of disc W
3) Reaction from corner of obstacle i.e R_B.

As disc turns over the corner B, reaction R_A will not exist. Hence C will be the point of concurrency for this three force system in equilibrium.

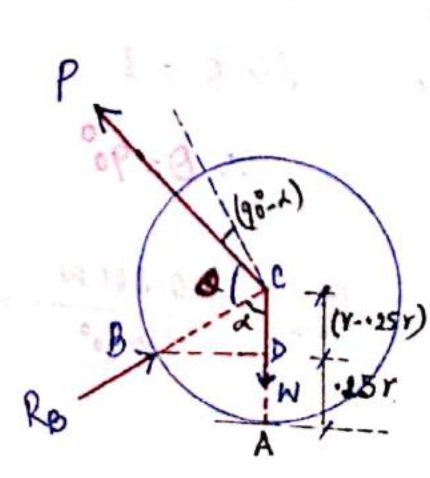


Fig. Ex. 2.A. 12(a)

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from geometry, Refer Fig. Ex. 2. A. 12 (a)

Draw a horizontal line BD from B and join BC.

$$AD = 0.25\,r$$

$$BC = r$$

$$CD = (r - 0.25r) = 0.75r$$

In ΔBCD; $\cos\alpha = \frac{CD}{BC} = \frac{0.75r}{r}$

$$\therefore \alpha = 41.41^\circ$$

Also; $BD = \sqrt{(BC)^2 - (CD)^2} = \sqrt{r^2 - (0.75r)^2}$

$$\therefore BD = 0.661\,r$$

Consider $\Sigma M_B = 0$

$\therefore W \times BD - P\sin\theta \times BC = 0$ .... resolving P at C.

$$\therefore W \times (0.661\,r) = P\sin\theta \times r$$

$$\therefore P = \frac{0.661\,W}{\sin\theta}$$

$\therefore$ For P to be minimum, $\sin\theta$ must be maximum.

i.e $\sin\theta = 1$

$\therefore \theta = 90^\circ$ ..... Ans

and; $P_{min} = \frac{0.661\,W}{\sin 90^\circ} = 0.661\,W$ .... Ans.

Problems based on Equilibrium of bars

Ex. 2.A.21. A light bar AB of length 'L' rests against two smooth inclined surfaces as shown in Fig. Ex. 2.A.21. A Vertical force 'p' acts at point C such that AC = a and CB = b. Determine angle θ defining equilibrium in terms of a, b and β.

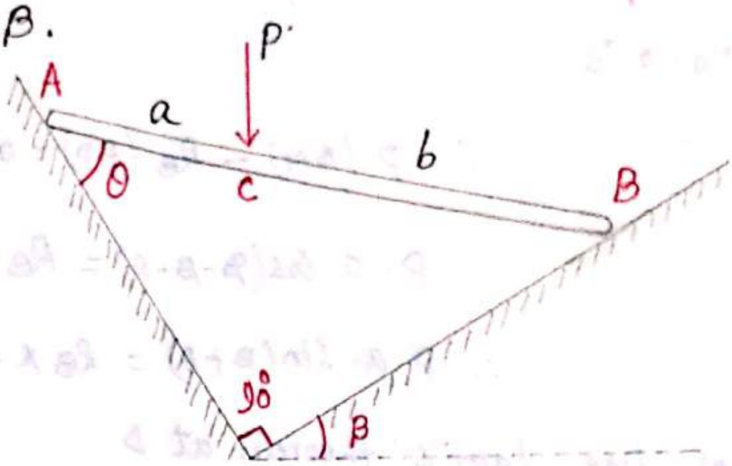

Fig. Ex. 2.A.21

Soln: observe the number of forces acting:

Here bar is subjected to three forces i.e

1) Normal reaction at A ; R_A
2) Normal reaction at B ; R_B
3) vertical force P

Draw F.B.D of bar:

Since the bar is in equilibrium under the action of three non-parallel forces, they must be concurrent. Refer Fig. Ex. 2.A.21(a)

Geometry :

In ΔAOB; $\sin\theta = \frac{OB}{AB}$ $\therefore$ $OB = L\sin\theta$

$\therefore$ $AD = L\sin\theta$

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In Δ AMC;

$$\cos(90-\beta-\theta) = \frac{AM}{AC}$$

$$\therefore AM = a\cos(90-\beta-\theta)$$

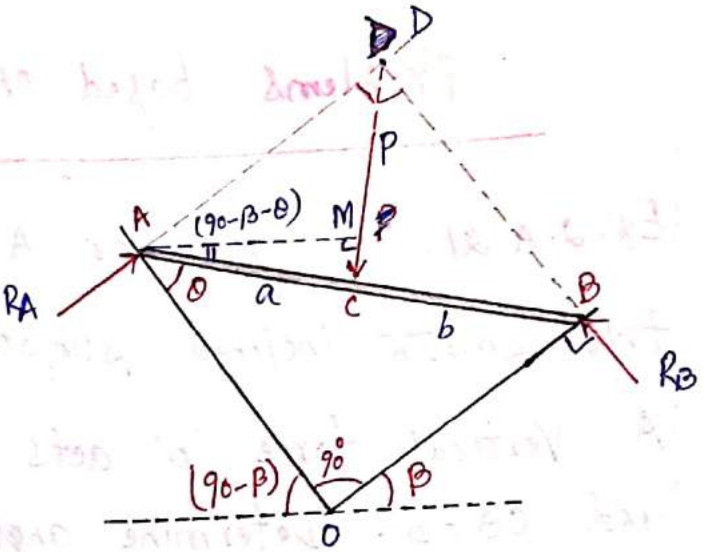

Fig. Ex. 2.A.21(a)

Conditions of Equilibrium:

$\Sigma M_A = 0$ (+↻)

$$\therefore P(AM) - R_B(AD) = 0$$

$$\therefore P\cdot a\cos(90-\beta-\theta) = R_B \times L\sin\theta$$

$$\therefore P\cdot a\cdot\sin(\beta+\theta) = R_B \times L\sin\theta \quad ---(i)$$

Now, Use Lami's theorem at D

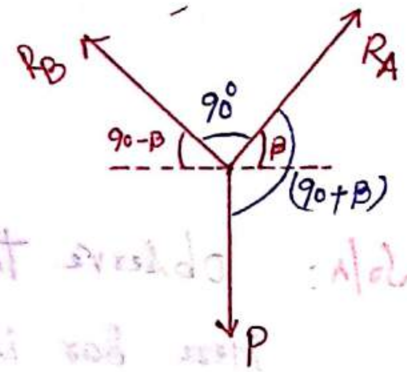

Fig. Ex. 2.A.21(b)

$$\therefore \frac{P}{\sin 90°} = \frac{R_B}{\sin(90+\beta)}$$

$$\therefore R_B = \frac{P\cos\beta}{\sin 90°} = P\cos\beta$$

Substituting this value in equation (i), we get

$$P\cdot a\cdot\sin(\beta+\theta) = P\cos\beta\cdot L\sin\theta$$

$$\therefore a(\sin\beta\cos\theta + \cos\beta\cdot\sin\theta) = (a+b)\sin\theta\cdot\cos\beta$$

$$\therefore a\sin\beta\cdot\cos\theta + \cancel{a\cos\beta\sin\theta} = \cancel{a\sin\theta\cos\beta} + b\sin\theta\cdot\cos\beta$$

$$\therefore b\sin\theta\cdot\cos\beta = a\sin\beta\cdot\cos\theta$$

$$\tan\theta = \frac{a}{b}\tan\beta$$

$$\therefore \boxed{\theta = \tan^{-1}\left(\frac{a\tan\beta}{b}\right)} \quad ---\ \text{Ans.}$$

Ex. 2.A.22 : A uniform bar AB of length L and weight W lies in a vertical plane with its ends resting on two smooth surfaces OA and OB. Find angle θ for equilibrium of bar. Refer. Fig. Ex. 2.A.22.

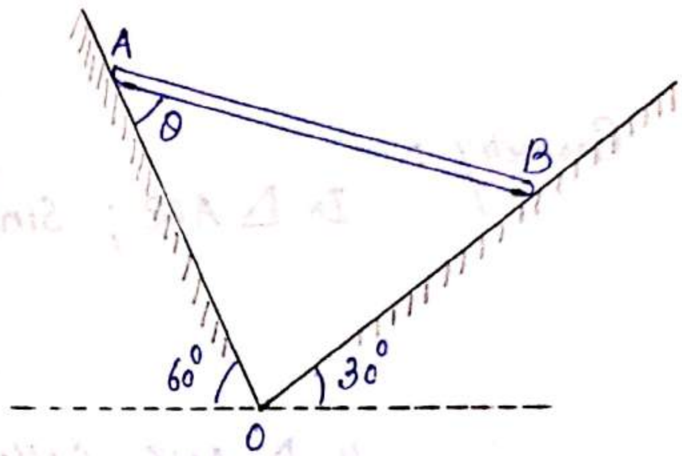

Fig : Ex. 2.A.22

Soln :

Observe the number of forces acting :

Here, bar is subjected to three forces i.e.

1) Normal reaction at A ; R_A
2) Normal reaction at B ; R_B
3) Self weight of bar, W.

Draw F.B.D. of bar :

Since the bar is in equilibrium under the action of three non-parallel forces, they must be concurrent. Refer Fig. Ex. 2.A.22(b)

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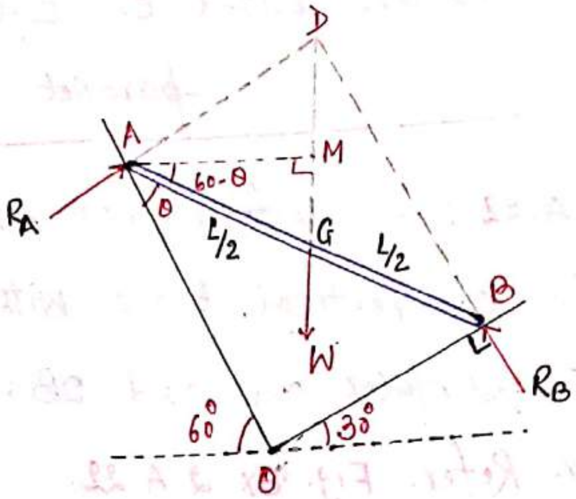


Fig. EX. 2. A. 22(a)

Geometry :

In ΔAOB ; $\sin\theta = \frac{OB}{AB}$ $\therefore OB = L\sin\theta$

$\therefore AD = L\sin\theta$

In ΔAMG ; $\cos(60-\theta) = \frac{AM}{AG}$ $\therefore AM = \frac{L}{2}\cos(60-\theta)$

Conditions of equilibrium:

$\Sigma M_A = 0$ +↻ $\therefore W(AM) - R_B(AD) = 0$

$$\therefore W \times \frac{L}{2}\cos(60-\theta) = R_B \times L\sin\theta$$

$$\therefore R_B \sin\theta = \frac{W}{2}\cos(60-\theta) \quad \text{------} (1)$$

Now, Use Lami's theorem at D

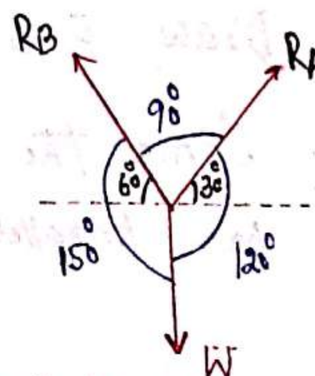


Fig. EX. 2. A. 22(b)

$$\therefore \frac{R_A}{\sin 150^\circ} = \frac{R_B}{\sin 120^\circ} = \frac{W}{\sin 90^\circ}$$

$$\therefore R_B = \frac{W \sin 120^\circ}{\sin 90^\circ} = 0.87\, W$$

Substituting this value in equation (i), we get

$$0.87\, W \sin\theta = \frac{W}{2} \cos(60-\theta)$$

$$\therefore 1.74 \sin\theta = \cos 60^\circ \cdot \cos\theta + \sin 60^\circ \cdot \sin\theta$$

$$\therefore \cancel{0.866}\ 0.874 \sin\theta = 0.5 \cos\theta$$

$$\therefore \tan\theta = \frac{0.5}{0.874}$$

$$\therefore \theta = 29.77^\circ \quad \text{---- Ans.}$$

Ex. 2.A.23 : A bar AB 10 m. long rests in horizontal position as shown in Fig. Ex. 2.A.23. on two smooth planes. Find distance x at which a load $P = 100$ N is to be placed to keep the bar in equilibrium. Neglect weight of bar.

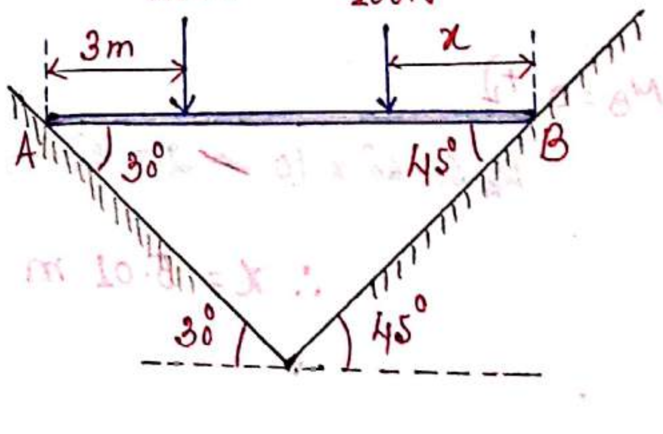


Fig. Ex. 2.A.23.

Soln : Draw F.B.D. of bar AB : Refer Fig. Ex. 2. A. 23(b).

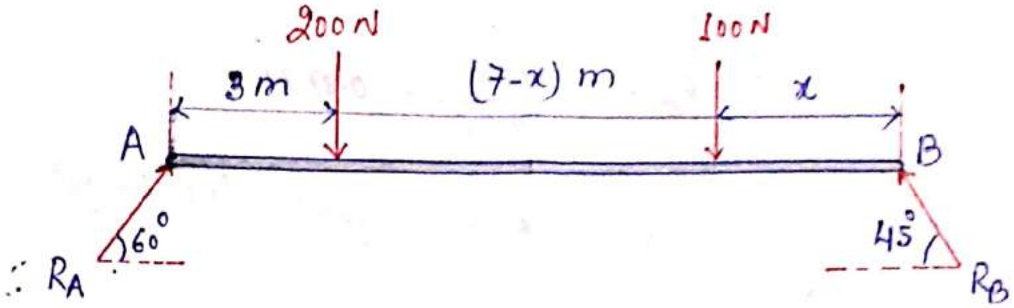


Fig. Ex. 2. A. 23(b)

Conditions of equilibrium :

$\underset{+}{\rightarrow} \Sigma F_H = 0$ $\quad R_A \cos 60^\circ - R_B \cos 45^\circ = 0$

$\therefore R_A = 1.414\, R_B$ ---- (i)

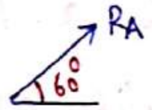
$+\!\uparrow \Sigma F_V = 0$ $\quad R_A \sin 60^\circ + R_B \sin 45^\circ - 200 - 100 = 0$

Substituting $R_A = 1.414\, R_B$ from equation (i)

$\therefore 1.414\, R_B \sin 60^\circ + R_B \sin 45^\circ = 300$

$\therefore R_B = \cancel{1.5}\ 155.3\ N$

∴ from equation (i) $\quad \therefore R_A = 219.6\ N$



$\Sigma M_B = 0 \; +\!\curvearrowright$

$R_A \sin 60^\circ \times 10 + 200 \times 7 - 100 \times x = 0$

$\therefore x = 5.01\ m$ ----- **Ans.**

Problems based on General Equilibrium

Ex. 2.A.32 : Three bars in one plane, hinged at their ends as shown in Fig. Ex.2.A.32 are submitted to the action of a force P = 50 N applied at the hinged B. Determine the magnitude of the force Q that it will be necessary to apply at the hinge C in order to keep the system of bars in equilibrium.

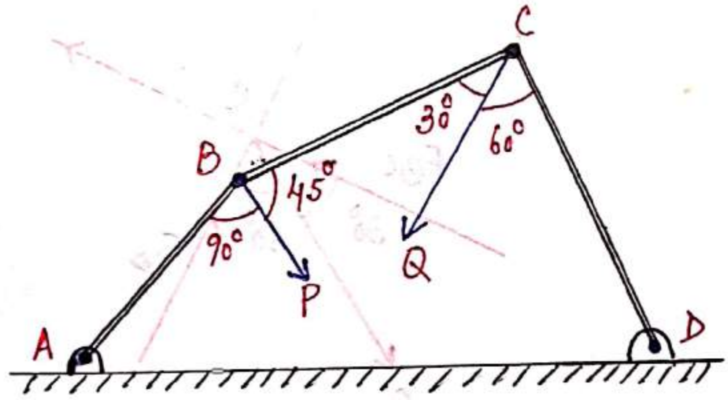


Fig. Ex.2.A.32

Soln :

Since AB and BC are hinged at B and inward force is acting at B, the bars will be subjected to compressive forces.

Draw F.B.D. of joint B :

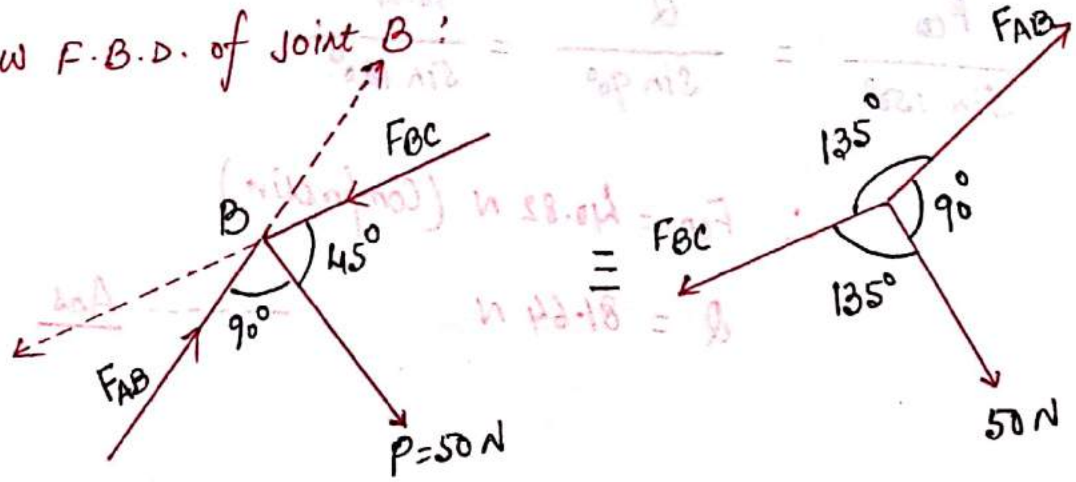


Fig. Ex.2.A.32(a)

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By Lami's theorem;

$$\frac{F_{AB}}{\sin 135^\circ} = \frac{F_{BC}}{\sin 90^\circ} = \frac{50}{\sin 135^\circ}$$

$\therefore$ $F_{AB} = 50\ N$ (Compressive)

$F_{BC} = 70.71\ N$ (Compressive)

Now, Consider F.B.D. of joint C :

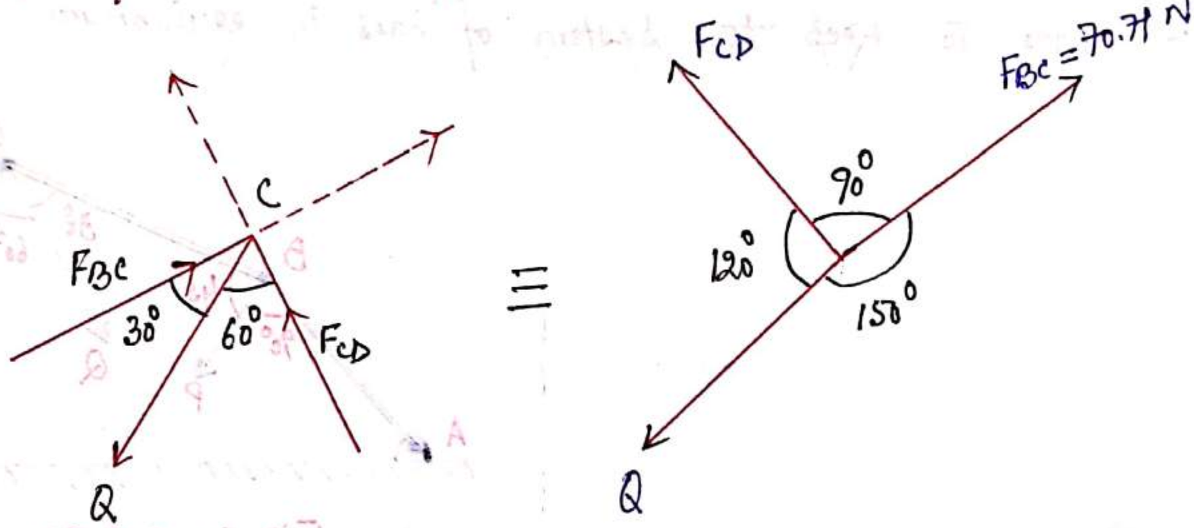

Fig: Ex. 2. A. 32(b)

By Lami's theorem;

$$\frac{F_{CD}}{\sin 150^\circ} = \frac{Q}{\sin 90^\circ} = \frac{70.71}{\sin 120^\circ}$$

$\therefore$ $F_{CD} = 40.82\ N$ (Compressive)

$Q = 81.64\ N$ ------ Ans.

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EX. 2. A. 33

Prob: 18. Find the tensile force in cables AB and CB shown in fig. The remaining cables ride over frictionless pulleys E and F. Refer Fig. EX. 2. A. 33.

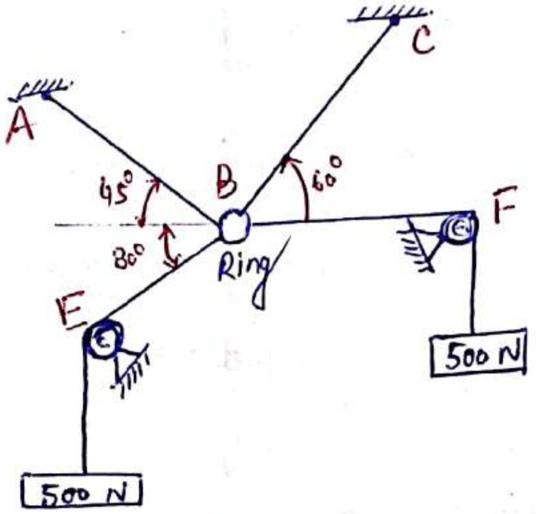
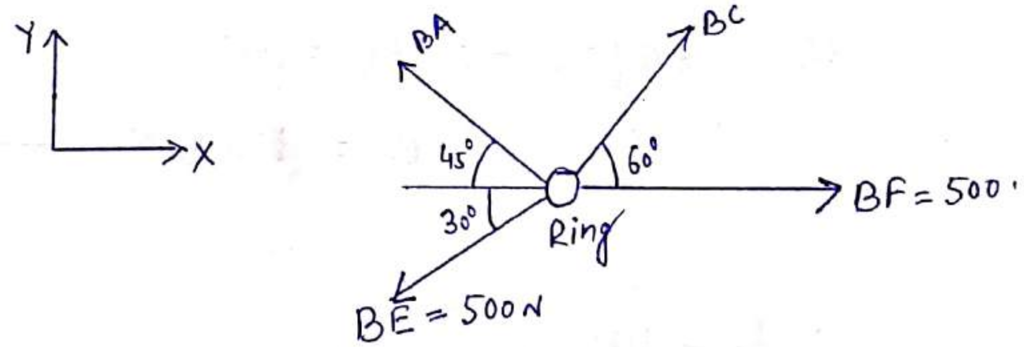


Fig. EX 2. A. 33

Solⁿ: Draw F.B.D of ring;



F.B.D (Fig: EX. 2. A. 33(a))

From equⁿ of equilibrium;

$$\Sigma F_x = 0$$

$$BF + BC\cos 60^\circ - BA\cos 45^\circ - BE\cos 30^\circ = 0$$

or, $500 + \frac{1}{2} BC - 0.707\, BA - 500\cos 30^\circ = 0$

or, $0.5\,BC - 0.707\,BA = 500\cos 30^\circ - 500$

$\therefore\ 0.5\,BC - 0.707\,BA = -66.98$ ———— (i)

$\Sigma F_y = 0,$

$BC \sin 60^\circ + BA \sin 45^\circ - BE \sin 30^\circ = 0$

or, $0.866\,BC + 0.707\,BA = BE \sin 30^\circ = 500 \sin 30^\circ$

$\therefore\ 0.866\,BC + 0.707\,BA = 250$ ———— (ii)

from (i) + (ii)

$0.5\,BC - 0.707\,BA + 0.707\,BA + 0.866\,BC = -66.98 + 250$

or, $BC(0.5 + 0.866) = 183$

$\therefore\ BC = 133.96\ N$ ———— (i)

$\therefore$ from (i),

$0.5\,BC - 0.707\,BA = -66.98$

or, $0.707\,BA = 0.5\,BC + 66.98 = 0.5 \times 133.96 + 66.98$

or, $0.707\,BA = 133.96$

$BA = 189.4\ N$ ———— (ii)

$$\left.\begin{aligned} T_{BC} &= 133.96\ N, \\ T_{BA} &= 189.4\ N; \end{aligned}\right\}\ \text{Ans.}$$

Ex. 2.A.34:

Prob: Find the force transmitted by wire BC shown in fig. The pulley E can be assumed to be frictionless in this problem. Refer Fig. Ex. 2.A.34

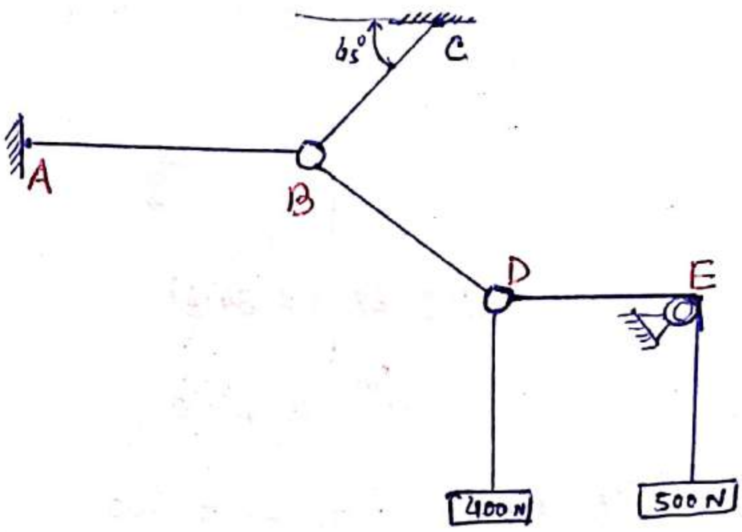


Fig. Ex. 2.A.34

Soln: From free body diagram of 'D'.

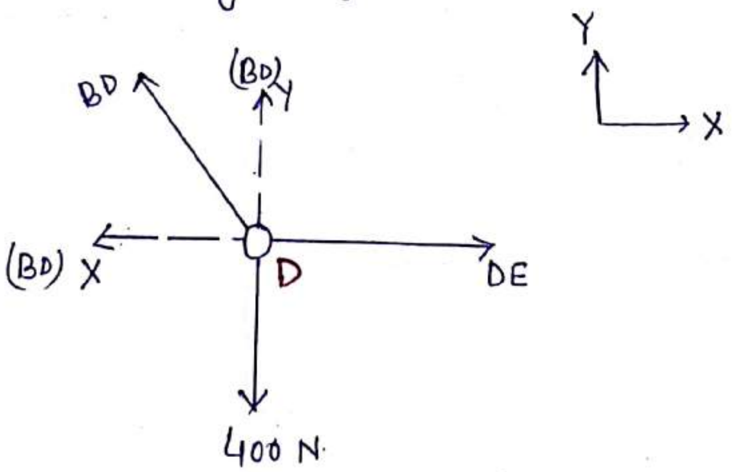


Fig. Ex. 2.A.34 (a)

$\Sigma F_x = 0;$ $(BD)_x = DE = 500$

$\therefore (BD)_x = 500\ N$ ——— (i)

$\Sigma F_y = 0;$ $(BD)_y = 400\ N$ ——— (ii)

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from free body diagram of 'B'

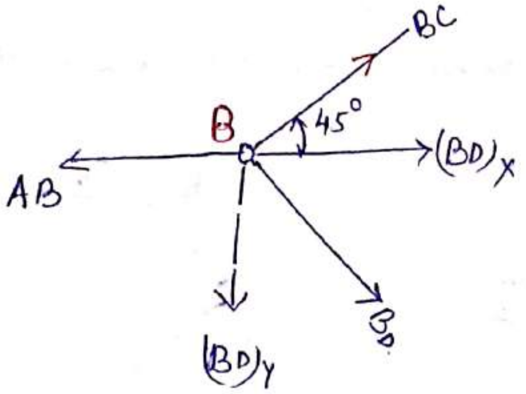

Fig. Ex. 2.A.34 (b)

$\Sigma F_y = 0;$

$$BC \sin 45^\circ - BD_y = 0$$

or, $BC \sin 45^\circ = BD_y = 400$

$$\therefore BC = 565.6 \text{ N} \quad \text{———} \quad (iii)$$

Hence; Tension in the wire BC is 565.6 N.

Ans.

Ex. 2.A.35 : A T-shaped bracket as shown in Fig. Ex. 2.A.35. is supported by a roller at E and small pegs at C and D. Neglecting friction, determine reactions at C, D and E.

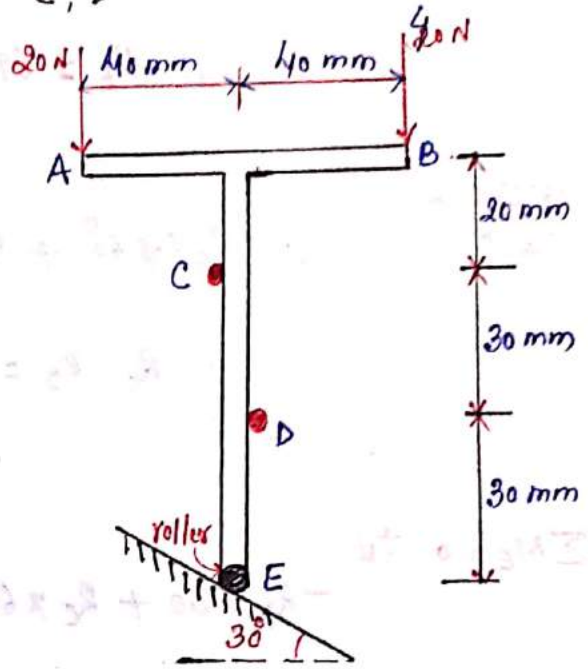

Fig. Ex. 2.A.35

Soln : Draw F.B.D. of T-shaped bracket.

R_E will be normal to inclined surface while R_C and R_D will be horizontal with assumed directions. Refer Fig. Ex. 2.A.35(a).

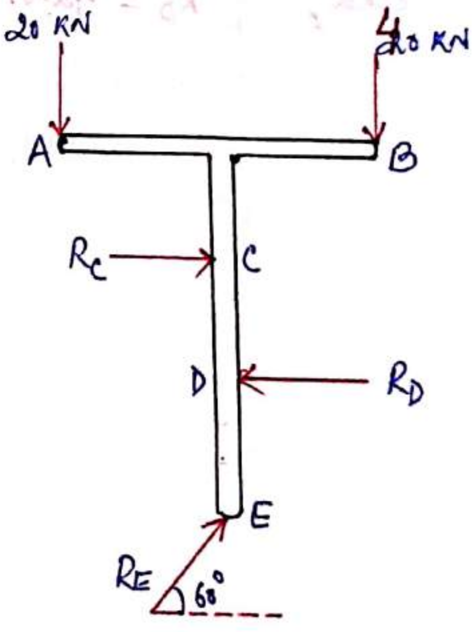

Fig. Ex. 2.A.35(a)

1.

$+\uparrow \Sigma F_V = 0$

$$\therefore \quad -20 - 40 + R_E \sin 60^\circ = 0$$

$$\therefore \quad R_E = 69.28 \ N$$

$\Sigma F_H = 0 \quad \xrightarrow{+}$

$$\therefore \quad R_E \cos 60^\circ + R_C - R_D = 0$$

$$\therefore \quad R_C - R_D = -34.64 \ N \qquad ---(i)$$

$\Sigma M_E = 0 \quad +\circlearrowright$

$$-R_D \times 30 + R_C \times 60 + 40 \times 40 - 20 \times 40 = 0$$

$$2R_C - R_D = -26.67 \qquad ----(ii)$$

Solving (i) and (ii), we get

$$\therefore R_C = 7.97 \ N \rightarrow$$

Ans.

and; $\quad R_D = 42.61 \ N \leftarrow$

Ex. 2.A.36 : A vertical pole ABCD is hinged at its base A and carries loads as shown in Fig. Ex. 2.A.36. (all in one plane). Determine (i) the magnitude and sense of horizontal force applied at D which would be necessary for equilibrium, and ii) the resultant reaction at A.

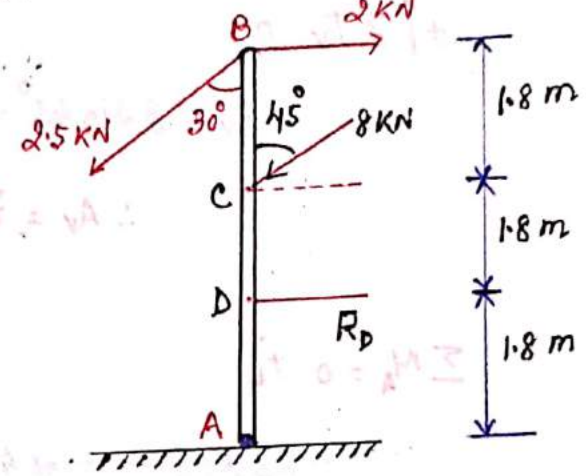

Fig. Ex. 2. A. 36.

Soln : Draw F.B.D. of vertical pole ABCD;
A_H and A_V will be normal at point A and R_D will be horizontal with assumed directions. Refer Fig. Ex. 2. A. 36(a)

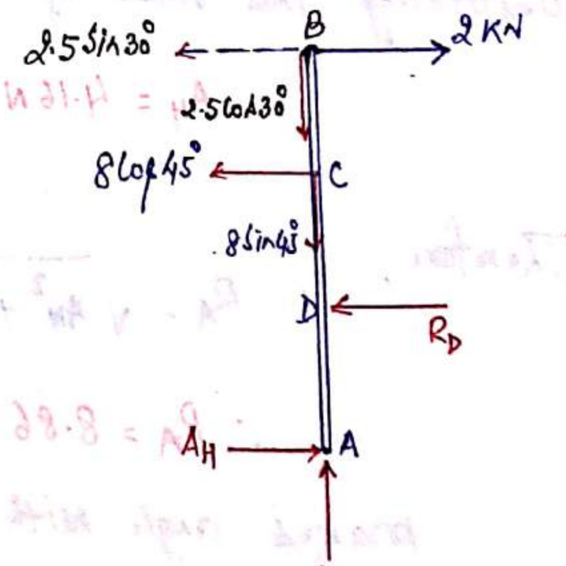

Fig. Ex. 2. A. 36(a)

Now,

$$\xrightarrow{+} \Sigma F_H = 0$$

$$A_H - R_D - 8\cos 45^\circ - 2.5\sin 30^\circ + 2 = 0$$

$$\therefore A_H - R_D = 4.90\ N \quad \text{-------} (i)$$

$$+\uparrow \Sigma F_V = 0$$

$$A_V - 8\sin 45^\circ - 2.5\cos 30^\circ = 0$$

$$\therefore A_V = 7.82\ N \uparrow$$

$$\Sigma M_A = 0 \ +\circlearrowright$$

$$-R_D \times 1.8 - 8\cos 45^\circ \times 3.6 - 2.5\sin 30^\circ \times 5.4 + 2 \times 5.4 = 0$$

$$\therefore R_D = -9.06\ N$$

$$R_D = 9.06\ N \rightarrow \quad \text{----} \ \underline{\text{Ans.}}$$

Substituting this value of R_D in equation (i), we get

$$A_H = 4.16\ N \leftarrow$$

Therefore,

$$R_A = \sqrt{A_H^2 + A_V^2} = \sqrt{4.16^2 + 7.82^2}$$

$$\therefore R_A = 8.86\ N$$

makes angle with horizontal

$$\text{i.e.} \quad \alpha = \tan^{-1}\left[\frac{A_V}{A_H}\right] = \tan^{-1}\left[\frac{7.82}{4.16}\right]$$

$$\alpha = 61.98^\circ$$

$$\therefore R_A = 8.86\ N \ \nwarrow 61.98^\circ \quad \text{---} \ \underline{\text{Ans.}}$$

Part B : Analysis of Beams.

2.9. Beam :

Usually beam is a horizontal straight member of a structure which transfer the load to the other beam or column etc. for safety and stability of the structure.

For a beam, simple support, roller support, hinged support and fixed support are possible. Each support provides some kind of freedom and some constraints. When freedom is provided, there is no reaction and as constraints are introduced, there is reaction.

2.9.1. Types of beam :

As per support specification the following are the different types of beam.

1. Simply supported beam.
2. Cantilever beam
3. propped cantilever beam
4. Fixed beam
5. overhanging beam
6. Beam with hinge and roller support
7. Continuous beam.

1) Simply Supported beam:

A beam which is just resting on the supports without any connection is called simply supported beam.

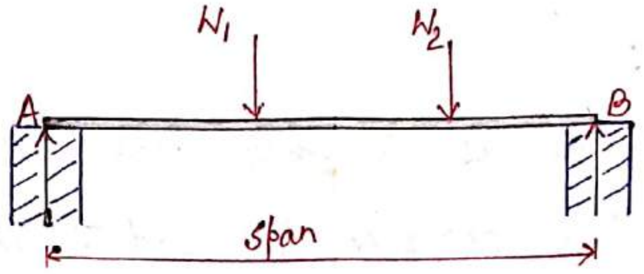

Fig: 5

2) Cantilever beam:

A beam which is fixed at one end and free at the other end is called cantilever beam.

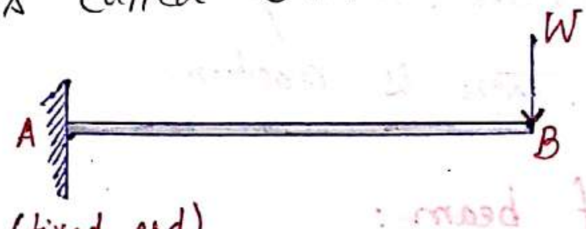

Fig: 6

3) propped Cantilever:

A beam which is fixed at one end and other end (or any intermediate point) is simple support is called propped cantilever beam.

Fig. 7.

4. Fixed beam :

A beam which is fixed at both end is called fixed beam.

Fig: 8

5. Overhanging beam :

A beam which is not supported at the ends but there may be intermediate support and part of the beam is extended beyound the support.

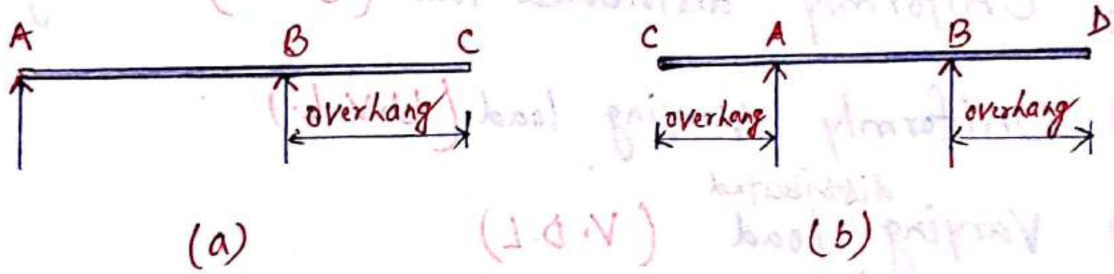

Fig. 9

6. Beam with hinge and roller support :

A beam which is hinged at one end and other is on roller is called hinged and roller supported beam.

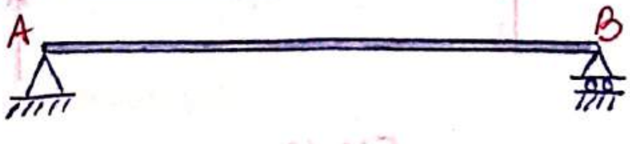

Fig. 10

7. Continuous beam : A beam which is supported on number of supports is called continuous beam.

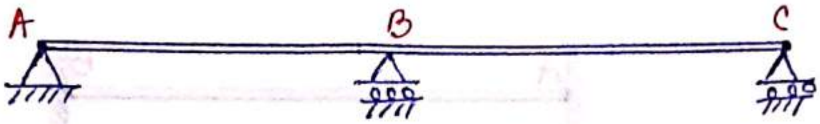

Fig. 11

2.9.2. Types of loads:

Generally, a beam carries following types of loading:

1). point load or concentrated load
2) uniformly distributed load (U.D.L) or rectangular load
3) Uniformly varying load. (U.V.L.)
4) Varying distributed load (V.D.L)

1). point load :

A load acting at a single point on the beam is called point load.

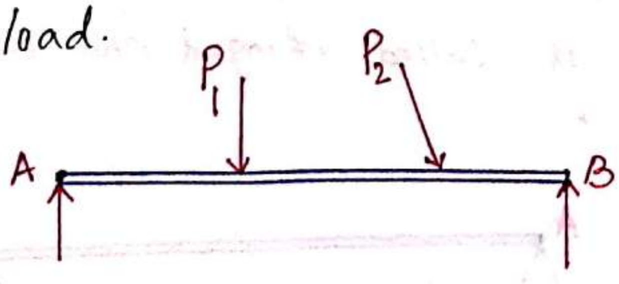

Fig. 12

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2) Uniformly distributed load :

A load which is spread over the beam (or part of beam) uniformly is called uniformly distributed load.

Uniformly distributed load can be converted into a point load by taking product of intensity of u.d.l. and spreading distance. This point load will be concentrate at the centre of u.d.l.

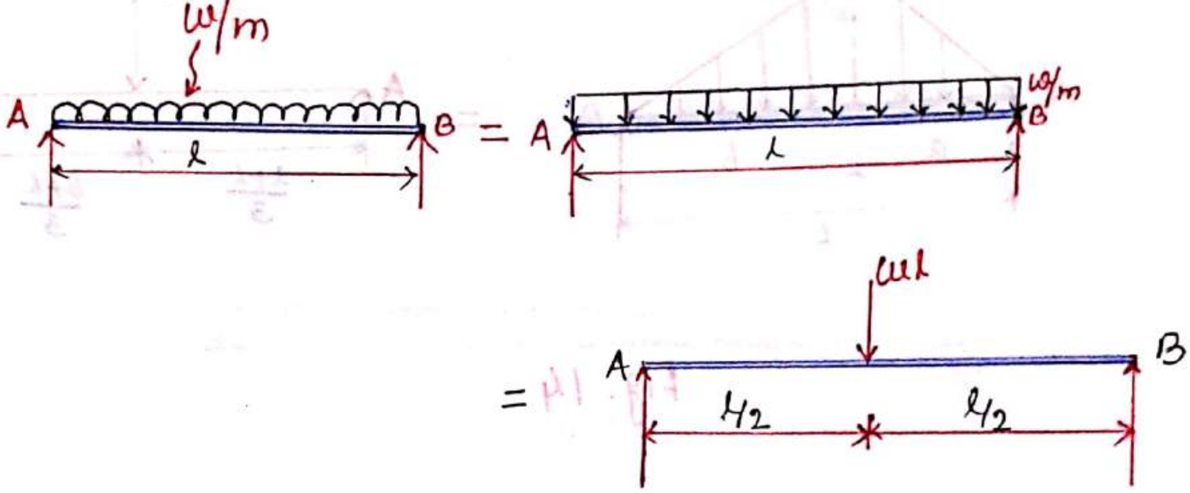
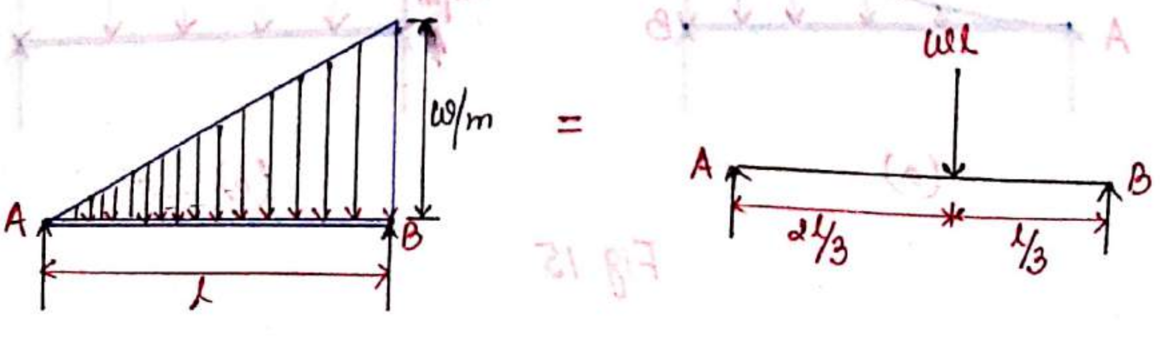


Fig. 13:

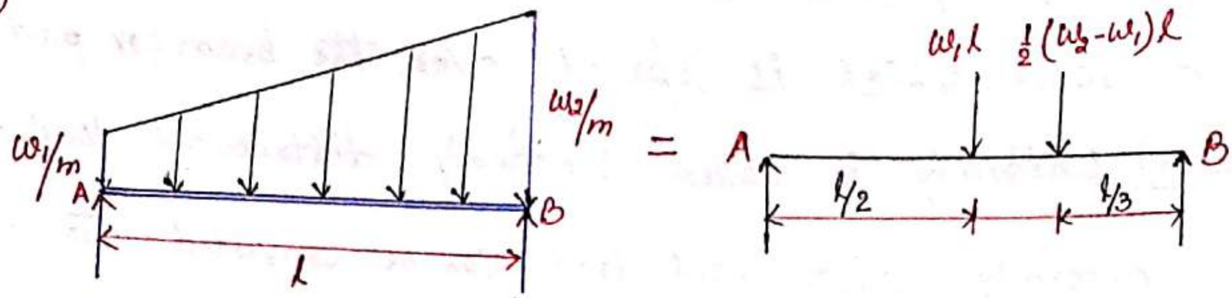
3) Uniformly varying load :

A load whose intensity is linearly varying between two points on the beam is known as U.V.L.

(a)



(b)



(c)

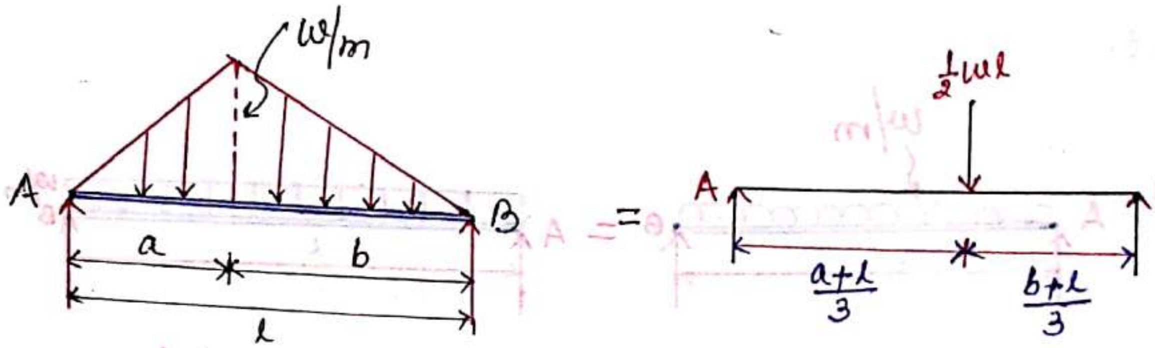


Fig. 14

4. Varying distributed load :

A load whose intensity is varying between two points on the beam is known as V.D.L.

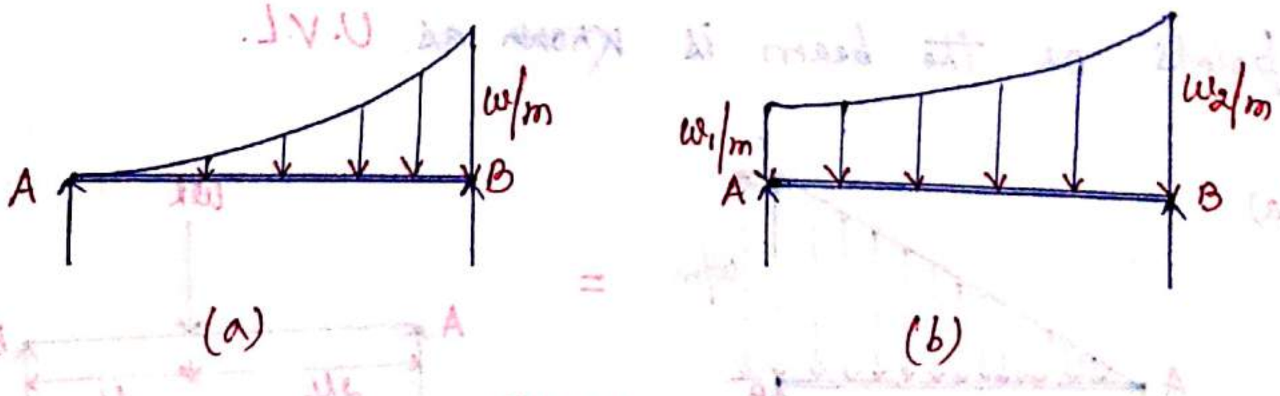


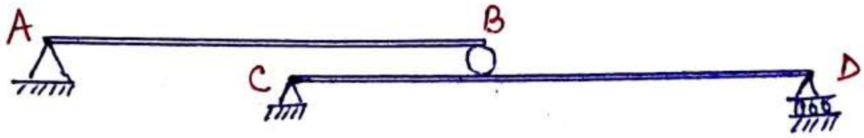
Fig. 15

~~2.9.~~

2.10 : Compound beam :

When two or more beams are joined together by means of internal hinge or internal roller, this combination is called as Compound beam.

(a)



(b)

Fig. 16

Problems based on Simple beam

Ex. 2.B.1 : Find the reactions at the supports for given beam as shown in Fig. Ex. 2.B.1.

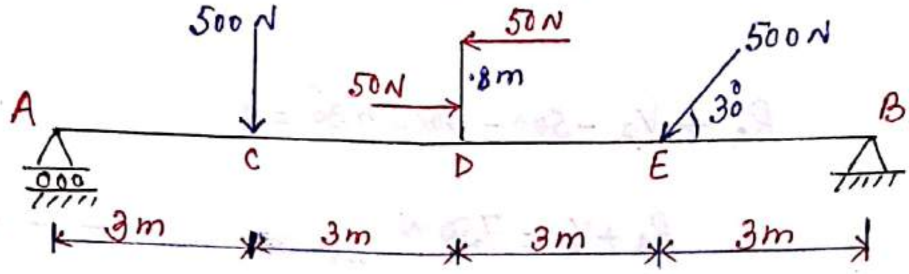


Fig. Ex. 2.B.1

Soln : Draw F.B.D of beam from the given description.

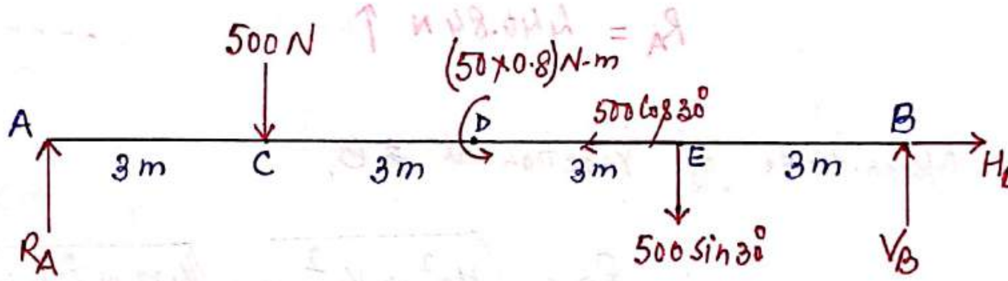


Fig. Ex. 2.B.1(a)

1. Two 50 N forces on a small bracket at D form an anticlockwise couple of $50 \times 0.8 = 40$ N-m
So, replacing these forces by the couple moment of 40 N-m ↺

2. Resolving 500 N inclined force into 500 sin 30° ↓ and 500 cos 30° ←

Now, applying conditions of equilibrium;

$\underset{\xrightarrow{+}}{\Sigma F_H} = 0 \qquad H_B - 500\cos 30^\circ = 0$

$\therefore H_B = 433.01\ N \rightarrow$

$+\uparrow \Sigma F_V = 0 \qquad R_A + V_B - 500 - 500\sin 30^\circ = 0$

$R_A + V_B = 750\ N \qquad \text{-----} \ (i)$

$\Sigma M_A = 0 \ +\circlearrowright; \qquad 500 \times 3 - 40 + 500\sin 30^\circ \times 9 - V_B \times 12 = 0$

$\therefore V_B = 309.16\ N \uparrow$

Substitute the value of V_B in equation (i), we get

$R_A = 440.84\ N \uparrow \qquad \text{----} \ \text{Ans.}$

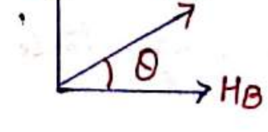
$\therefore$ Magnitude of reaction at ~~A~~ B;

$\therefore R_B = \sqrt{H_B^2 + V_B^2} = \sqrt{(433.01)^2 + (309.16)^2}$

$\therefore R_B = 532.05\ N$

Direction

$\tan\theta = \dfrac{V_B}{H_B} \qquad \therefore \theta = 35.52^\circ \qquad \text{----} \ \text{Ans.}$



Ex 2.B.2: Find the reactions at support of the beam as shown in Fig. Ex.2.B.2.

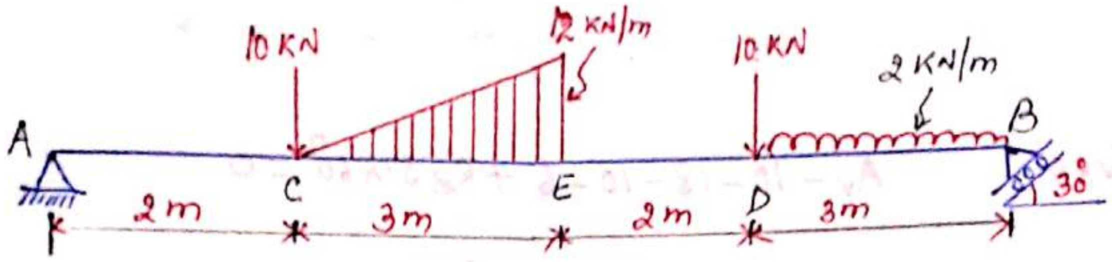


Fig. Ex.2.B.2

Soln: Draw F.B.D. of beam from the given description.

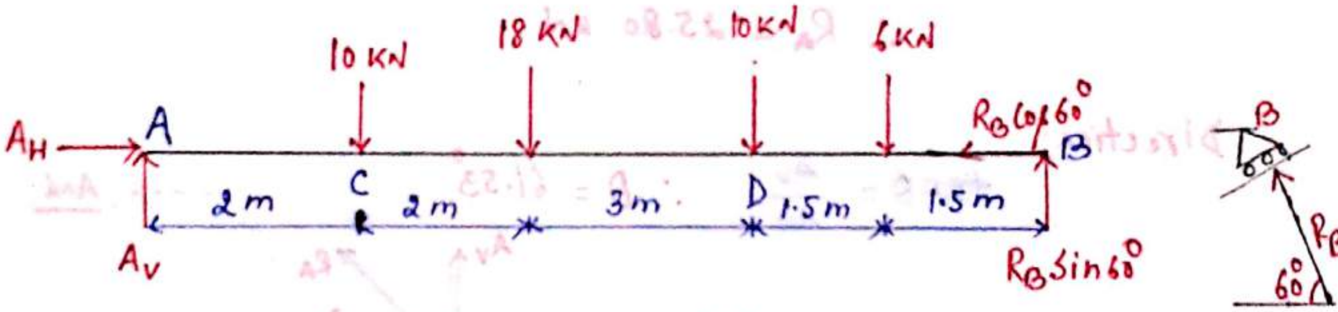


Fig. Ex.2.B.2(a)

1. The load between CE is triangular load = Area of triangle

$= \frac{1}{2} \times 3 \times 12 = 18$ KN ↓ acting at a

distance of $\frac{2}{3} \times 3 = 2$ m from C.

2. U.D.L between DB is equal to $2 \times 3 = 6$ KN ↓ acting at 1.5m from B.

Now, applying conditions of equilibrium;

$\Sigma M_A = 0$ +↻

$\therefore \quad 10 \times 2 + 18 \times 4 + 10 \times 7 + 6 \times 8.5 - R_B \sin 60° \times 10 = 0$

$\therefore R_B = 24.60\ KN$  ..... Ans.

$\xrightarrow[+]{\Sigma F_H = 0}$

$\therefore A_H - R_B \cos 60^\circ = 0$

$\therefore A_H = 24.60 \cos 60^\circ = 12.3\ KN \rightarrow$

$+\uparrow \Sigma F_V = 0 \qquad A_V - 10 - 18 - 10 - 6 + R_B \sin 60^\circ = 0$

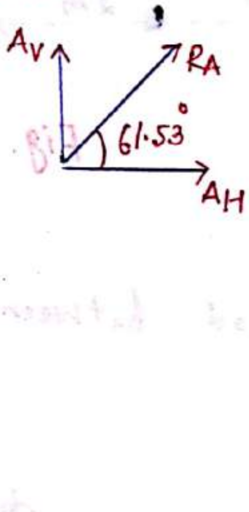
$A_V = 22.69\ KN \uparrow$

$\therefore$ Magnitude of reaction at A;

$$R_A = \sqrt{(A_H)^2 + (A_V)^2} = \sqrt{(12.3)^2 + (22.69)^2}$$

$\therefore R_A = 25.80\ KN$

Direction; $\tan\theta = \dfrac{A_V}{A_H} \quad \therefore \theta = 61.53^\circ$ ..... Ans.



Ex. 2.B.3: Find reaction components at the supports for the following beam as shown in Fig. Ex 2.B.3.

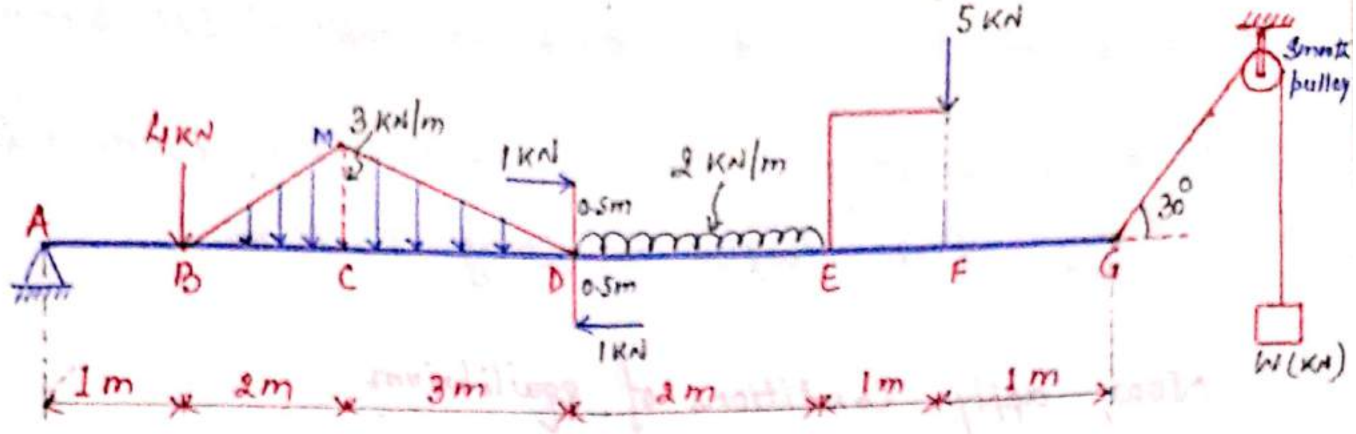


Fig. Ex. 2.B.3

Soln:

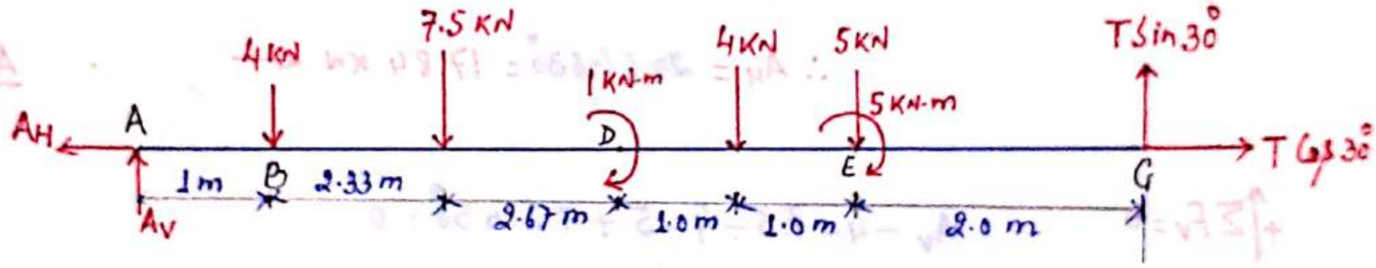


Fig. Ex. 2.B.3(a)

F.B.D. of beam involves

1) A triangular load over BCD

$$\text{load} = \text{Area of } \Delta \, BMD$$

$$= \frac{1}{2} \times 5 \times 3 = 7.5 \text{ KN} \downarrow \text{ acting at } \frac{2+5}{3} \left(\text{i.e. } \frac{a+l}{3}\right)$$

$$= 2.33 \text{ m from } B.$$

2) At D, a vertical bracket is subjected to a couple, the moment of which is $1 \times 1 = 1$ KN-m ↻

3) At E, an angle bracket is attatched. At E, a force-couple system is formed at E, which consists of a downward force of 5 kN and a clockwise couple of $5 \times 1 = 5$ kN-m.

4) At G, a string is connected which is passing over a smooth pulley, hence tension in string $T = W$ kN.

Now, apply conditions of equilibrium

$\Sigma M_A = 0$ $+\circlearrowright$;

$$4 \times 1 + 7.5 \times 3.33 + 1 + 4 \times 7 + 5 \times 8 + 5 - T \sin 30^\circ \times 10 = 0$$

$$\therefore T = 20.6 \text{ kN} = W$$

$\Sigma F_H = 0$ $\xrightarrow{+}$ $\quad -A_H + T \cos 30^\circ = 0$

$$\therefore A_H = 20.6 \cos 30^\circ = 17.84 \text{ kN} \leftarrow$$ --- Ans.

$+\uparrow \Sigma F_V = 0$ $\quad A_V - 4 - 7.5 - 4 - 5 + T \sin 30^\circ = 0$

$$A_V = 10.2 \text{ kN} \uparrow$$ --- Ans.

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Ex. 2.B.4 : Determine the intensity of distributed load w at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B. Refer Fig. Ex.2.B.4.

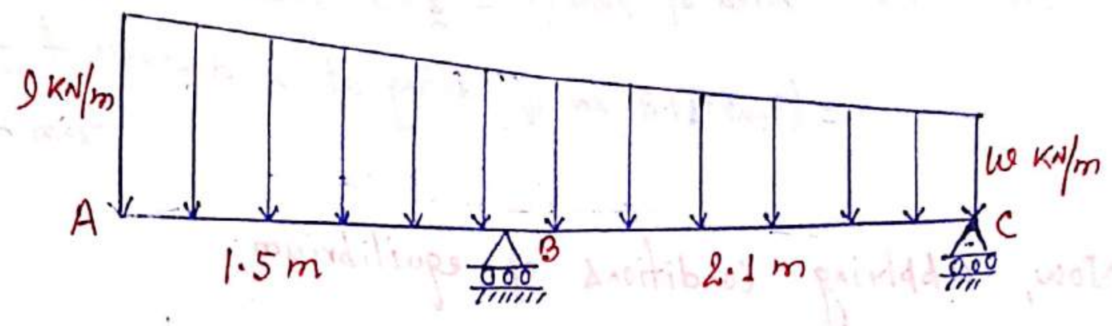

Fig. Ex. 2. B.4

Soln: Dividing given U.V.L into two parts as shown in Fig. Ex.2.B.4(a)

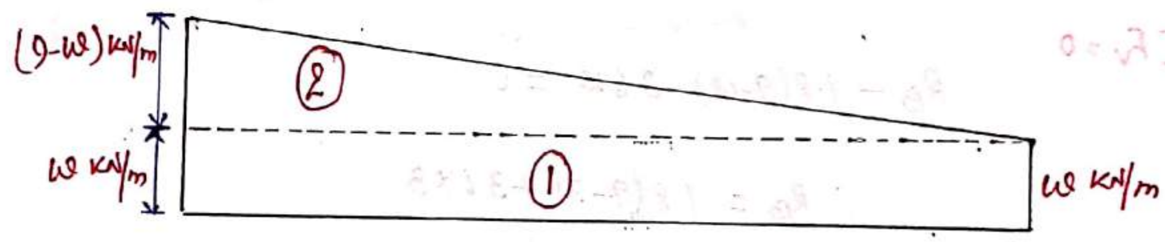

Fig. Ex.2.B.4(a)

~~Draw~~. F.B.D. of beam involves

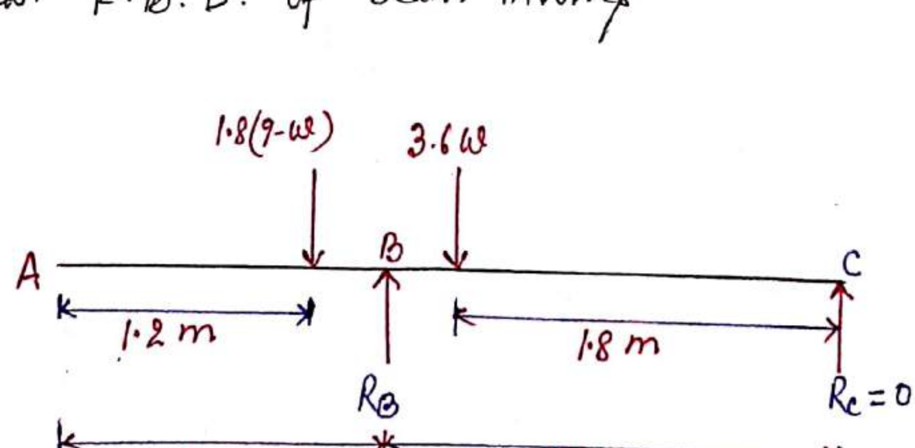

Fig. Ex.2.B.4(b)

1. Load W_1 = Area of part (1) = $3.6 \times w$

$= 3.6w$ kN ↓ acting at a distance of $\frac{1}{2} \times 3.6 = 1.8$ m from C

2. Load W_2 = Area of part (2) = $\frac{1}{2} \times (9-w) \times 3.6$

$= (9-w) \times 1.8$ kN ↓ acting at a distance of $\frac{1}{3} \times 3.6 = 1.2$ m from A.

Now, applying conditions of equilibrium;

$\sum M_B = 0$ +↻

$\therefore \quad 3.6w \times 0.3 - 1.8(9-w) \times 0.3 = 0$

$\therefore w = 3$ kN/m --- Ans.

+↑ $\sum F_V = 0$

$R_B - 1.8(9-w) - 3.6w = 0$

$\therefore R_B = 1.8(9-3) + 3.6 \times 3$

$\therefore R_B = 21.6$ kN ↑ -- Ans.

Ex. 2.B.7 : A beam supports a load distributed parabolically over its length. Determine the resultant of this distributed load and its line of action. Also determine the support reaction. [Refer Fig. Ex. 2 B.7].

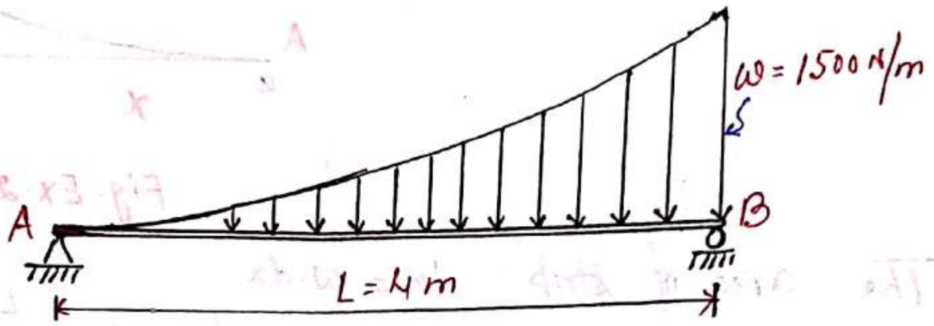

Fig. Ex. 2.B.7

Soln :

Here, given loading is a varying load so to find out total load and its point of application using integration method.

The equation of the parabolic load is of the form

$$x^2 = K\omega \quad \text{------ (1)}$$

Where K is a constant.

at $x = 4\,m$, $\omega = 1500\ N/m$

from (i), we get

$$K = \frac{x^2}{\omega} = \frac{4 \times 4}{1500} = \frac{16}{1500}$$

Resultant of distributed load w

Select an elementary strip of thickness dx as shown in Fig. Ex. 2. B.7(a).

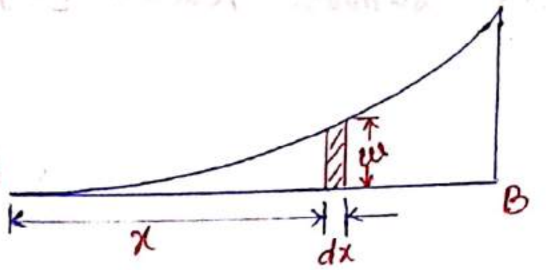

Fig. Ex. 2. B.7(a)

The area of strip $dW = w \cdot dx$

$\therefore$ Total area bounded $W = \int_0^L w\,dx = \int_0^L \frac{x^2}{K} \cdot dx = \left[\frac{x^3}{3K}\right]_0^L$

$$= \frac{L^3}{3K} = \frac{4 \times 4 \times 4}{3 \times 16/1500} = 2000\ N$$

point of application of resultant load W

The point of application of resultant load W is

$$\bar{x} = \frac{\int_0^L w x\,dx}{W} = \frac{\int_0^L \frac{x^2}{K} \cdot x \cdot dx}{W} = \frac{L^4}{4KW}$$

$$= \frac{4^4}{4 \times \frac{16}{1500} \times 2000} = 3\ m \text{ from } A.$$

F. B. D. of beam AB involves : (Ref. Fig. Ex. 2. B.7(b)).

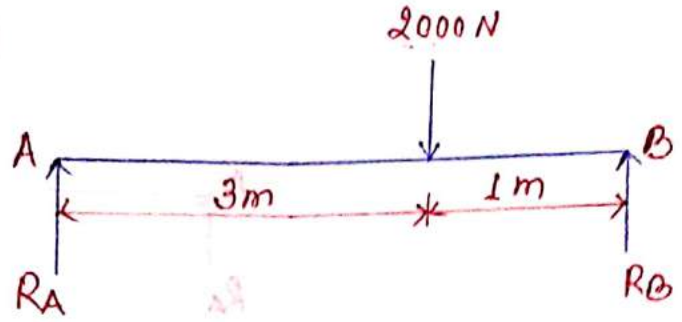

Fig. Ex. 2.B.7 (b)

Conditions of Equilibrium:

$\Sigma M_A = 0 \; +\circlearrowright$ $\quad 2000 \times 3 - R_B \times 4 = 0$

$\therefore R_B = 1500\ N \uparrow$ -- Ans.

$+\uparrow \Sigma F_y = 0$ $\quad R_A + R_B = 2000$

$\therefore R_A = 500\ N \uparrow$ --- Ans.

Important Note:

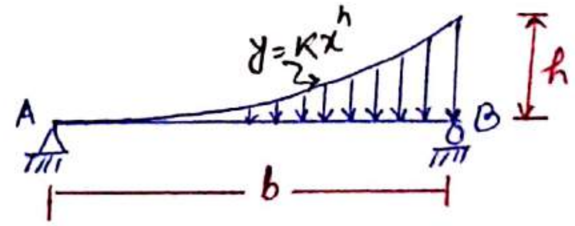

Fig: 17

Converting this varying load in to point load, we can use general formula:

$$\text{Area of diagram} = \frac{1}{(n+1)} \times b \times h = \text{total load}$$

and its line of action $= \frac{1}{(n+2)} \times b$ from point B.

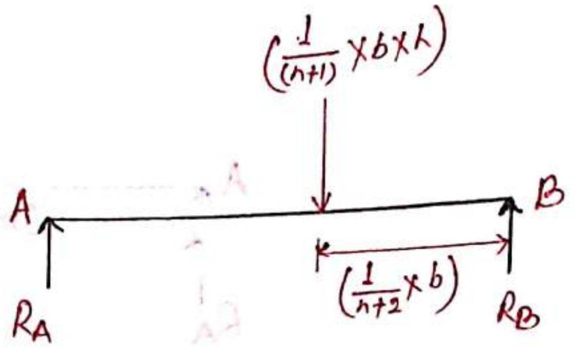
$\left(\frac{1}{(n+1)} \times b \times h\right)$
A
B
$\left(\frac{1}{n+2} \times b\right)$
R_A
R_B

Fig: 18

Problems based on Compound beam

There are following ~~for~~ procedure for the analysis of Compound beam.

Step 1 : Disconnect the beam from internal hinge or internal roller

Step 2 : Draw F.B.D. of each member separately by assuming internal reaction components as equal and opposite.

Step 3 : Apply conditions of equilibrium to each beam to find out reaction components.

Ex. 2.B.8 : Determine reactions at A, B and C for given beam as shown in Fig. Ex. 2B.8.

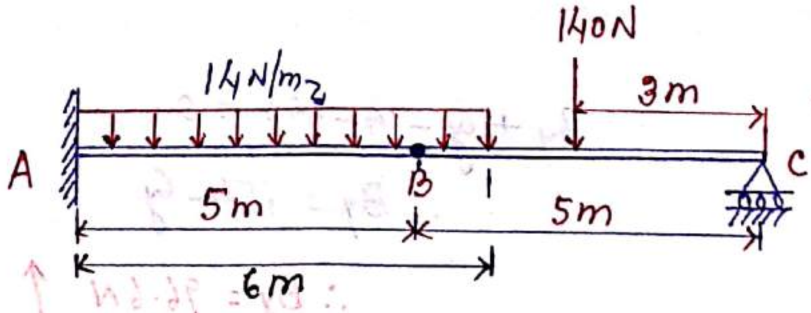

Fig. Ex. 2.B.8

Soln : F. B. D. of beams AB and BC

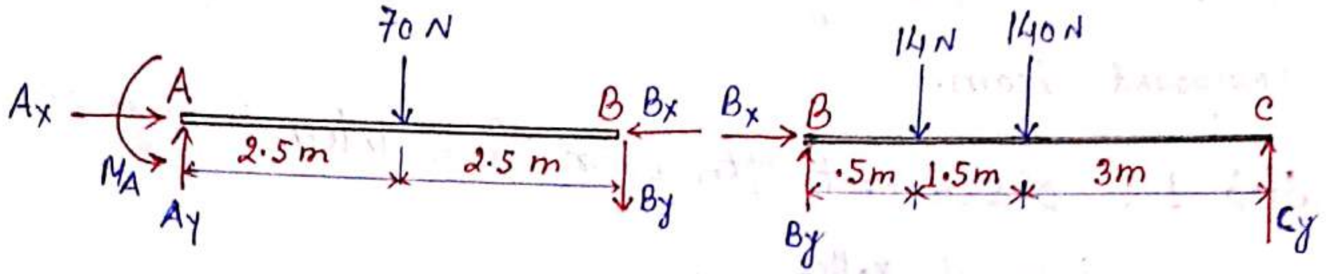

Fig. Ex. 2.B.8 (a)

<u>Consider beam BC :</u>

Conditions of equilibrium :

$\underset{+}{\xrightarrow{\Sigma F_x = 0}}$; $B_x = 0$; since beam carries no force in horizontal direction.

$\Sigma M_B = 0$ + ;

$$14 \times 0.5 + 140 \times 2 - C_y \times 5 = 0$$

$$\therefore C_y = 57.4 \text{ N} \uparrow$$ --- Ans.

$+\uparrow \Sigma F_y = 0$

$$\therefore B_y + C_y - 14 - 140 = 0$$

$$\therefore B_y = 154 - C_y$$

$$\therefore B_y = 96.6 \text{ N} \uparrow$$ --- <u>Ans</u>.

Consider beam AB :

Conditions of equilibrium

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x - B_x = 0$

$A_x - 0 = 0 \qquad \therefore A_x = 0$

$+\uparrow \Sigma F_y = 0 \qquad \therefore A_y - B_y - 70 = 0$

$A_y = 70 + B_y$

$\therefore A_y = 166.6 \text{ N} \uparrow$ --- Ans.

$\Sigma M_A = 0 \; +\circlearrowright$

$-M_A + 70 \times 2.5 + B_y \times 5 = 0$

$\therefore M_A = 70 \times 2.5 + 96.6 \times 5$

$\therefore M_A = 658 \text{ N-m} \circlearrowleft$ --- Ans.

Important observation:

If a horizontal compound beam carries no horizontal force or any horizontal component of a force, all horizontal reaction components must be equal to zero.

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Ex. 2.B.9 : Find reactions at A, B, C and D for given beam as shown in Fig. Ex. 2.B.9.

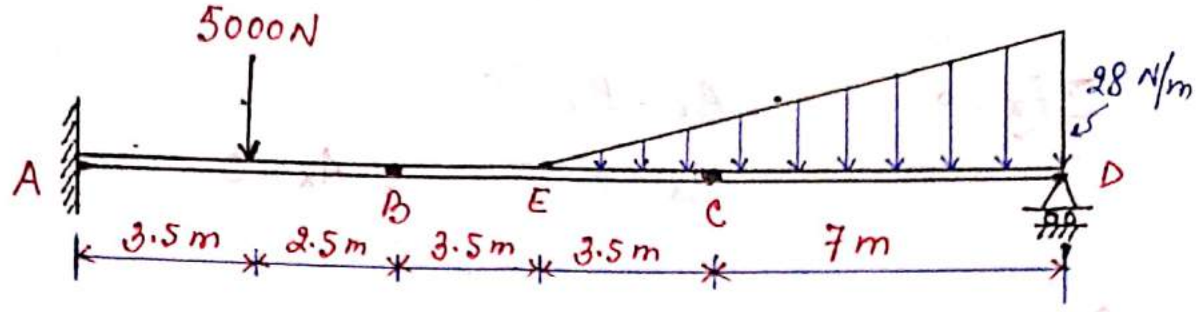

Fig. Ex. 2.B.9

Soln :

The triangular load is acting on both beams, the load distribution must be shown in Fig. Ex. 2.B.9 (a)

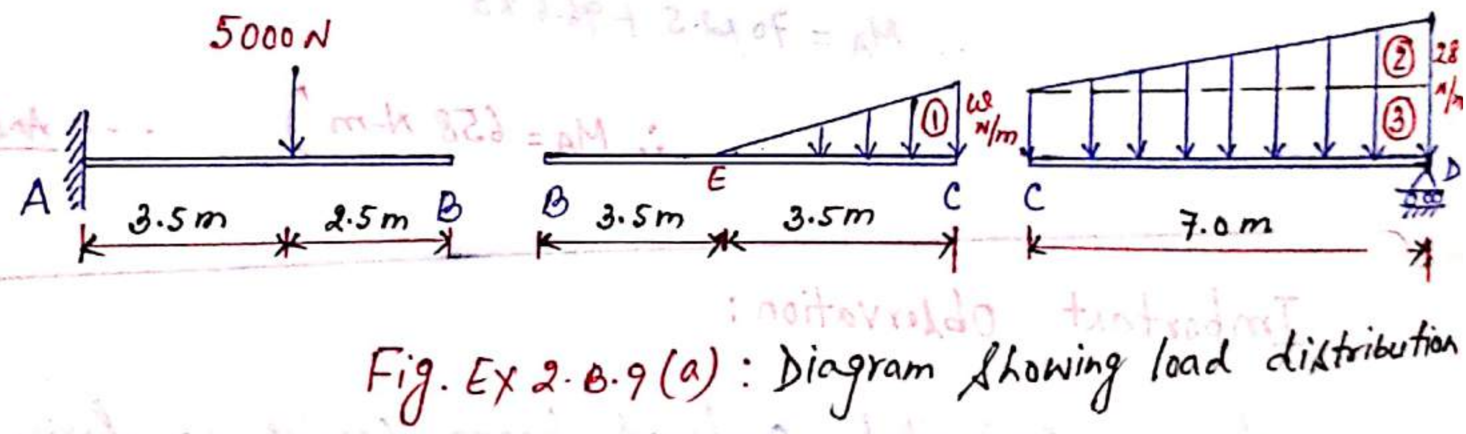

Fig. Ex 2.B.9 (a) : Diagram showing load distribution

Let ω be the intensity of U.V.L at C.

For two similar triangles (Ref. Fig. Ex. 2.B.9(b)).

$$\frac{28}{10.5} = \frac{\omega}{3.5}$$

$$\therefore \omega = 9.34 \text{ N/m}$$

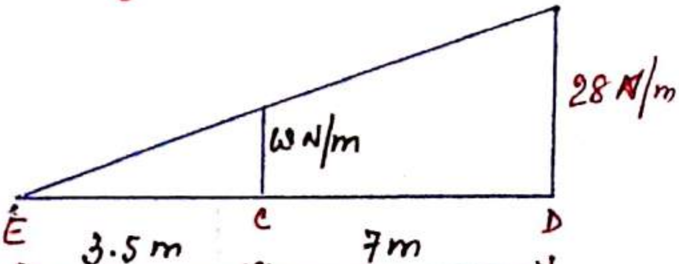

Fig. Ex. 2.B.9 (b)

Draw F.B.D. of beam AB, BC and CD

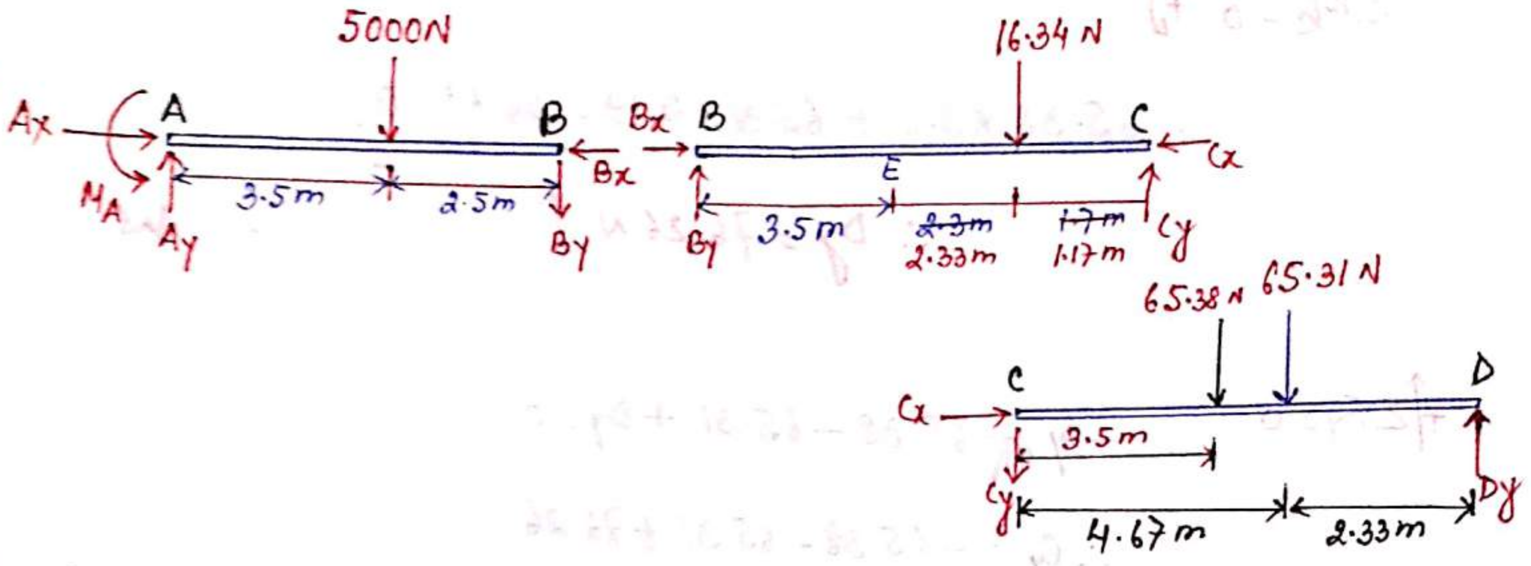

Fig. Ex. 2.B.9 (c)

Now,

Load W_1 = Area of part 1 $= \frac{1}{2} \times 3.5 \times 9.34$

$= 16.34$ N $\downarrow$ acting at $\frac{1}{3} \times 3.5 = 1.17$ m from C

Load W_2 = Area of part 2 $= \frac{1}{2} \times 7 \times (28 - \omega) = \frac{1}{2} \times 7 \times (28 - 9.34)$

$= 65.31$ N $\downarrow$ acting at $\frac{2}{3} \times 7 = 4.67$ m from C

Load W_3 = Area of part 3 $= \omega \times 7 = 9.34 \times 7$

$= 65.38$ N $\downarrow$ acting at $\frac{7}{2} = 3.5$ m from C.

Now, no horizontal force acting on the system,

therefore $A_x = B_x = C_x = 0$

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Consider beam BCD :

$\Sigma M_C = 0 \; +\circlearrowright$

$$\therefore 65.38 \times 3.5 + 65.31 \times 4.67 - D_y \times 7 = 0$$

$$\therefore D_y = 76.26 \text{ N} \uparrow$$ --- Ans.

$+\uparrow \Sigma F_y = 0$ $\quad -C_y - 65.38 - 65.31 + D_y = 0$

$$\therefore C_y = -65.38 - 65.31 + 76.26$$

$C_y = -54.43$ N (Negative sign indicates that assumed direction ~~is~~ of C_y is wrong)

$$\therefore C_y = 54.43 \text{N} \uparrow$$ --- Ans.

Consider beam BC :

$\cancel{\Sigma M_B} = 0 \; +\circlearrowright$

~~16.34 × 5.83~~

$+\uparrow \Sigma F_y = 0$ $\quad B_y + C_y - 16.34 = 0$

$$\therefore B_y + (-54.43) - 16.34 = 0$$

$$\therefore B_y = 70.77 \text{ N} \uparrow$$ ---- Ans.

Consider beam AB:

$\Sigma M_A = 0 \; +\circlearrowright$

$$-M_A + 5000 \times 3.5 + B_y \times 6 = 0$$

$$\therefore M_A = 5000 \times 3.5 + 70.77 \times 6$$

$$M_A = 17924.62 \, N \circlearrowright \qquad \text{---- Ans.}$$

$+\uparrow \Sigma F_y = 0$

$$A_y - B_y - 5000 = 0$$

$$\therefore A_y = B_y + 5000$$

$$\therefore A_y = 5070.77 \, N \uparrow \qquad \text{---- Ans.}$$

Ex. 2.B.10: Find reaction components at the internal hinge B and supports A and C for the compound beam. Refer Fig. Ex. 2.B.10.

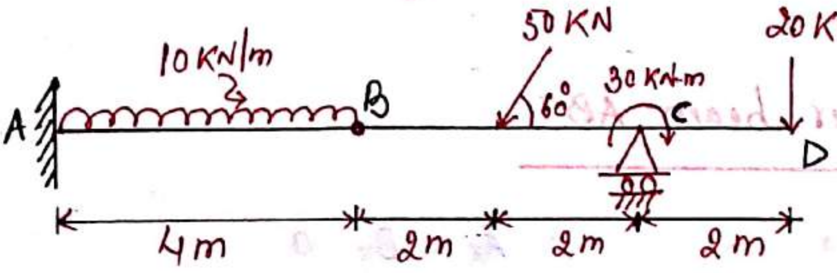


Fig. Ex. 2.B.10

Soln: Draw F.B.D of beam AB and BD (Fig. Ex. 2.B.10 (a)).

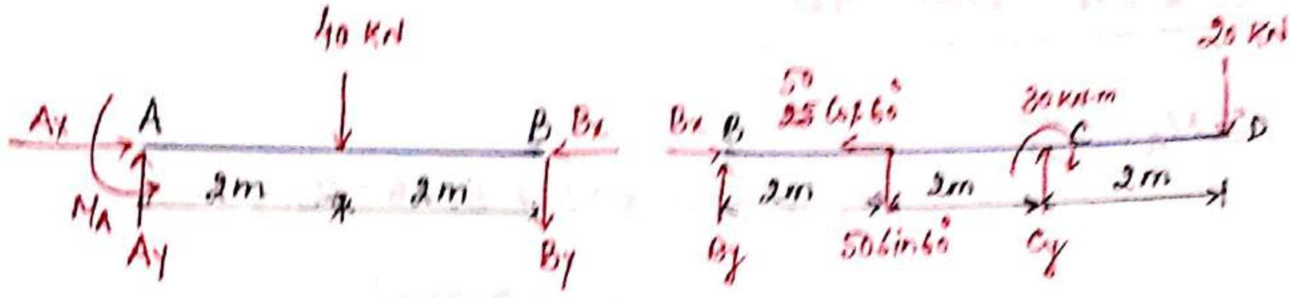


Fig. Ex. 2.B.10(a)

Consider beam BD :

$\xrightarrow[+]{\sum F_x = 0}$ $\qquad B_x - \overset{50}{\cancel{25}} \cos 60° = 0$

$\therefore B_x = 25 \text{ KN } (\rightarrow)$ --- Ans

$\sum M_B = 0 \; +\circlearrowright;\qquad 50 \sin 60° \times 2 + 30 + 20 \times 6 - C_y \times 4 = 0$

$\therefore C_y = 59.15 \text{ KN } \uparrow$ ---- Ans

$+\uparrow \sum F_y = 0 \qquad B_y + C_y - 50 \sin 60° - 20 = 0$

$\therefore B_y = 4.15 \text{ KN } \uparrow$ -- Ans

Consider beam AB :

$\xrightarrow[+]{\sum F_x = 0} \qquad A_x - B_x = 0$

$\therefore A_x = 25 \text{ KN } (\rightarrow)$ --- Ans.

$\sum M_A = 0 \; +\circlearrowright; \qquad -M_A + 40 \times 2 + B_y \times 4 = 0$

$\therefore M_A = 96.6 \text{ KN-m} \circlearrowright$ --- Ans.

$+\uparrow \sum F_y = 0$

$$A_y - 40 - B_y = 0$$

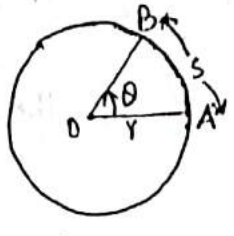
$$\therefore A_y = 44.15 \text{ KN} \uparrow$$ ----- Ans.

Rotation of rigid bodies

Angular motion:

The displacement of the body in rotation is measured in terms of angular displacement θ, where θ is in radians.

When a particle in a body moves from position A to B, the displacement is θ as shown in figure.



i.e $\boxed{\theta = \frac{s}{r}}$

Angular velocity : The rate of change of angular displacement with time is called angular velocity and is denoted by ω.

$$\boxed{\omega = \frac{d\theta}{dt}}$$

Angular acceleration: The rate of change of angular velocity with time is called angular acceleration and is denoted by α.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The angular accel^n may be expressed in another form;

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\boxed{\alpha = \omega \frac{d\omega}{d\theta}}$$

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Relationship between angular ~~accel~~ motion and linear motion

When the particle moves from A to B, the distance travelled by it is S.

i.e $\boxed{S = r\theta}$

The tangential velocity of the particle is called as Linear velocity and is denoted by V, then

$$\boxed{V = \frac{ds}{dt} = r\frac{d\theta}{dt}}$$

The linear acceleration of the particle in the tangential direction a_t is given by

$$\boxed{a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}}$$

* While treating the curvilinear motion, it has been shown that, if V is the tangential velocity, then there is a radial acceleration V^2/r.

Denoting radial acceleration by a_n, then

$$\boxed{a_n = \frac{V^2}{r} = r\omega^2}$$

Uniform angular velocity :

If the angular velocity is uniform, the angular distance moved in t seconds by a body having angular velocity ω radians/sec is given by

$$\boxed{\theta = \omega t \quad \text{radians}} \quad \text{i.e. } \omega = \frac{\theta}{t} \text{ rad/sec.}$$

* uniform angular velocity is characterised by zero angular acceleration.

Sometimes, the angular velocity is given in terms of number of revolution per minute.

Since, there are 2π radians in one revolution and 60 sec in one minute, then

$$\text{angular velocity } \omega = \frac{2\pi N}{60} \text{ rad/sec} \quad \text{Where } N = \text{rpm.}$$

Uniformly accelerated rotation :

Let us consider, the uniformly accelerated motion with angular acceleration α, then

$$\alpha = \frac{d\omega}{dt}$$

$$\text{or, } \omega = \alpha t + c_1$$

Where c_1 is constant of integration.

If the initial velocity is ω_0, then

$$\omega_0 = \alpha \times 0 + c_1 \Rightarrow c_1 = \omega_0$$

$$\therefore \boxed{\omega = \omega_0 + \alpha \cdot t}$$

Again from the definition of angular velocity

$$\omega = \frac{d\theta}{dt} = \omega_0 + \alpha t$$

$$\therefore \theta = \omega_0 t + \frac{1}{2}\alpha t^2 + C_2$$

Where C_2 = Constant of integration.

at $t = 0$, $\theta = 0$ $\Rightarrow$ $0 = 0 + 0 + C_2$

$$\Rightarrow C_2 = 0$$

$$\therefore \boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$$

Now,

$$\alpha = \omega \cdot \frac{d\omega}{d\theta}$$

or, $\alpha \cdot d\theta = \omega \, d\omega$

Integrating, we get

$$\alpha\theta = \frac{1}{2}\omega^2 + C_3$$

Where C_3 = Constant of integration.

Initially, $\theta = 0$ and $\omega = \omega_0$

Hence we get,

$$\alpha \times 0 = \frac{1}{2}\omega_0^2 + C_3$$

$$\Rightarrow C_3 = -\omega_0^2/2$$

$$\therefore \alpha\theta = \frac{\omega^2}{2} + \frac{-\omega_0^2}{2}$$

or, $\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$

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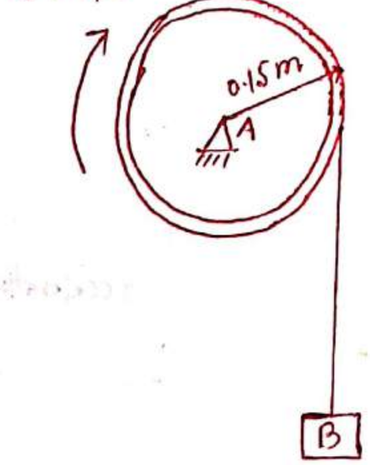
Thus for uniformly accelerated angular motion.

$$\left.\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \end{aligned}\right\}.$$

Compare with for uniformly accelerated linear motion

$$\left.\begin{aligned} V &= u + at \\ s &= ut + \tfrac{1}{2}at^2 \\ V^2 &= u^2 + 2as \end{aligned}\right\}.$$

(Prob): A motor gives ~~disc~~ flywheel A an angular accel^n of $\alpha_A = (0.6t^2 + 0.75)\ rad/s^2$, where t is in seconds. If the initial angular velocity is $\omega_0 = 6\ rad/sec$, Determine the magnitude of the velocity and accel^n of block B when $t = 2$ sec.



Sol^n: Given

$$\alpha_A = 0.6t^2 + 0.75$$

$$\omega_0 = 6\ rad/sec.$$

Now,

$$\alpha_A = 0.6t^2 + 0.75$$

$$\frac{d\omega}{dt} = 0.6t^2 + 0.75$$

$$\therefore\ \omega = 0.6\frac{t^3}{3} + 0.75t + c$$

At $t = 0$, $\omega = 6\ rad/sec$; $\therefore\ c = 6$

$$\therefore\ \omega = 0.2t^3 + 0.75t + 6$$

at $t = 2$ sec; $\omega = 9.1\ rad/s$

$\therefore$ Velocity $V = r\omega = 0.15 \times 9.1 = 1.365\ m/s$

$$\boxed{\text{Velocity } V = 1.365\ m/s}$$

and;

$$\alpha = 0.6t^2 + 0.75$$

$t = 2$ sec; $\quad \alpha = 3.15 \ rad/s^2$

$\therefore$ acceleration $= r\alpha$

$$= 0.15 \times 3.15$$

$$= 0.4725 \ m/s^2$$

$$\boxed{\text{acceleration } (a) = 0.4725 \ m/s^2}$$

Ans.

(Prob) 2: A flywheel increases its speed uniformly from 30 to 60 r.p.m in 10 secs. The diameter of the wheel is 3 m. Calculate (i) angular accelⁿ (ii) the no. of revolutions made by the wheel during 10 secs (iii) the linear accelⁿ of a point on the circumference of the wheel, at the end of these 10 secs.

Soluⁿ: Here, Initial angular speed

$$\omega_0 = 30 \text{ r.p.m} = 30 \times \frac{2\pi}{60} = \pi \text{ rad/sec.}$$

Angular speed at $t = 10$ sec is given by

$$\omega = 60 \text{ r.p.m} = 60 \times \frac{2\pi}{60} = 2\pi \text{ rad/s}$$

As the angular speed increases uniformly, the angular accleration is Constant.

$$\therefore \omega = \omega_0 + \alpha t \Rightarrow 2\pi = \pi + \alpha \times 10$$

$$\therefore \alpha = \frac{\pi}{10} = 0.314 \text{ rad/s}^2$$

Angular displacement in 10 sec;

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \theta = \pi \times 10 + \frac{1}{2} \times .314 \times 10^2$$

$$\theta = 47.12 \text{ radians}$$

$$\therefore \text{No. of revolutions made during 10 sec.} = \frac{47.12}{\cancel{10}\ 2\pi} = 7.5$$

Resultant linear accelⁿ $a = \sqrt{a_t^2 + a_n^2}$

where $a_t = r\alpha = 1.5 \times 0.314 = 0.471 \text{ m/s}^2$

$a_n = r\omega^2 = 1.5 \times (2\pi)^2 = 59.217 \text{ m/s}^2$

$$\boxed{a = 59.219 \text{ m/s}^2}$$

Virtual Work

Virtual work method is an alternative approach in
analysis of problems based on equilibrium. This method
provides a deeper insight into the analysis of mechanical
systems, mechanisms and interconnected bodies and enables
us to study the stability of systems in equilibrium.

Work done by a Force:

Consider a force P acting on the body as shown in fig......

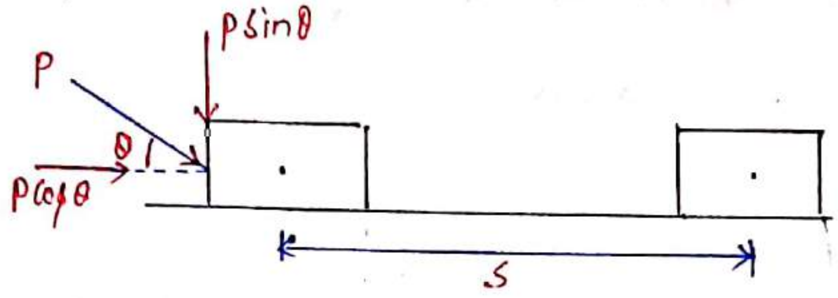


fig:

The workdone by force during displacement s is given by,

$$\text{Work done} = P\cos\theta \times s$$

* Work is positive, when the working component of the force is in the same direction as the displacement.
* Work is negative, when the working component of the force is in the opposite direction ~~as~~ to the displacement.
* No work is done, when the working component of the force $(P\sin\theta)$ is perpendicular to the displacement.

Work done by a Couple

When a couple M acts on the body, it ~~a~~ changes its angular position by an amount $d\theta$, the work done is given by

$$U = M \cdot d\theta$$

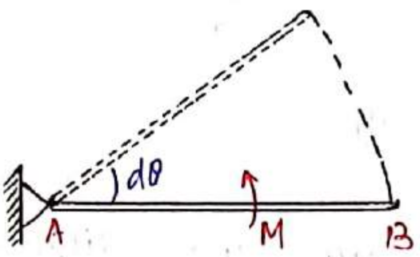


Fig:

* Work done by a couple is positive if M has the same sense as $d\theta$ and negative if M has a sense opposite to that of rotation.
* Sometimes if a couple tends to increase the angle θ, work done is considered as positive.

Concept of Virtual Work:

In the previous article, we have discussed that the work done by a force is equal to the force multiplied by the distance through which the body has moved in the direction of the force. But if the body is in equilibrium, under the action of a system of forces, the work done is zero. If we assume that the body, in equilibrium, undergoes an infinite small imaginary displacement (known as virtual displacement), some work will be imagined to the ~~to~~ done. Such an imaginary work is called virtual work. This concept of virtual work, is very useful in finding out the unknown forces in structures.

" If a system of forces or a system of bodies in equilibrium undergoes a small, arbitrary displacement (virtual displacement), some imaginary work is said to be done. This imaginary work done due to imaginary displacement and actual force or couple is called as virtual work".

Mathematically,

Virtual work = Virtual displacement X actual forces.

The term virtual indicates that the displacement does not really exist but only is assumed to exist.

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Principle of Virtual Work:

It states, "if a system of forces acting on a body or a system of bodies be in equilibrium and the system be imagined to undergo a small displacement consistent with the geometrical conditions, then the algebraic sum of the virtual works done by all the forces of the system is zero."

i.e. total virtual work done, $\delta U = 0$

Proof:

Consider a particle subjected to a force system and assume that particle undergoes a small displacement from A to A_1. If force system is balanced one, the actual displacement may not take place.

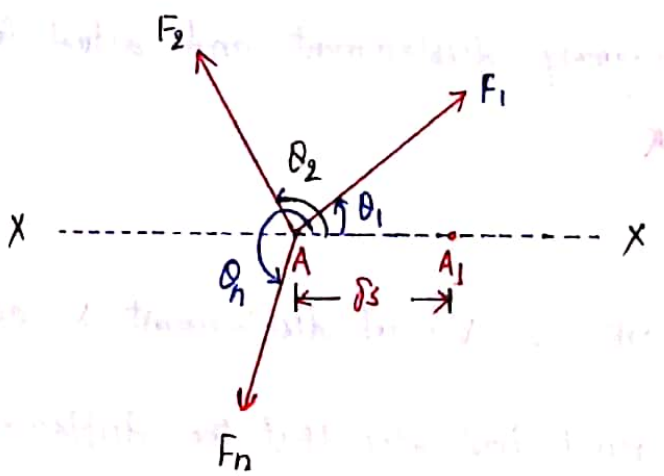

Fig:

Now, the workdone by force during imaginary displacement δs is given by

$$\delta U = F_1 \cdot \cos\theta_1 \cdot \delta s + F_2 \cos\theta_2 \cdot \delta s + \cdots\cdots F_n \cos\theta_n \cdot \delta s$$

$$\therefore \delta U = (F_1 \cos\theta_1 + F_2 \cos\theta_2 + \cdots\cdots F_n \cos\theta_n) \cdot \delta s$$

$$\delta U = \Sigma f_x \cdot \delta s$$

but if force system in equilibrium, resultant component Σf_x must be zero.

$$\therefore \boxed{\delta U = 0}$$

Conversely, virtual work may be stated as;

"of the algebric sum of the virtual work for every virtual displacement is zero, the system is in equilibrium."

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The following forces do not work and are omitted while applying the principle of virtual work:

(i) Forces normal to the direction of displacement;

(ii) reaction at smooth pins and hinges which do not move;

(iii) tension in a light inextensible string;

(iv) the internal forces of the nature of action and reaction between two bodies whose equilibrium is being considered together;

Sign Convention:

The following sign conventions are adopted while writing the expressions for virtual work:

(i) Upward force, forces acting towards right and the forces in the clockwise direction are considered positive.

(ii) Downward force, forces acting towards left and the forces in the anticlockwise direction are taken negative.

The virtual work becomes positive when the displacement is in the same direction as the force and negative when displacement is opposite to the direction of force.

Applications of the principle of Virtual work:

The principle of virtual work has very wide applications. But the following are important from the subject point of view:

1. Beams 2. Framed structures 3. Ladders

4. Lifting machine.

Application of the principle of virtual work on beams carrying point load.

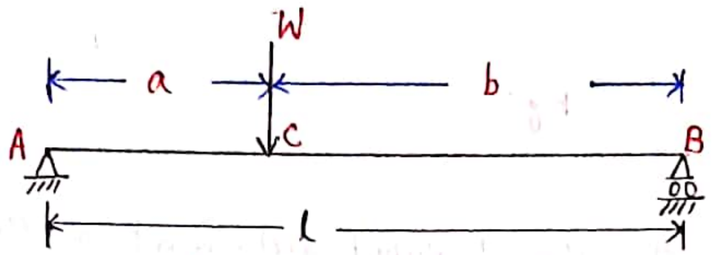

Fig:

Consider a beam AB, ~~simply~~ hinged and roller supported at its support and subjected to a point load W at C as shown in fig.....

Let R_A = reaction at A; and
~~R_A = reaction at A; and~~

R_B = reaction at B.

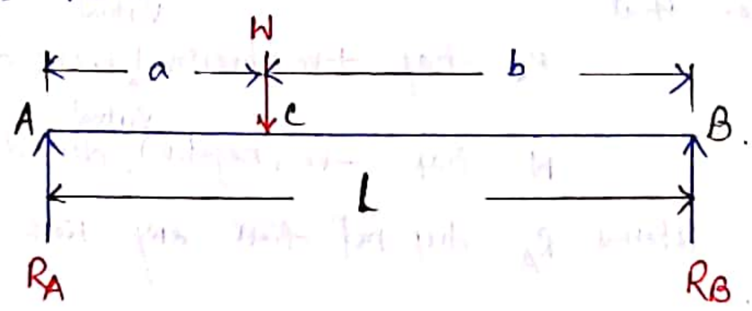

Fig:

Now, ~~beam AB is hinged at A~~. Consider an upward virtual displacement δ of the beam at B. This is due to the reaction at B acting upward as shown in fig......

Keeping A in its position, draw a displacement diagram

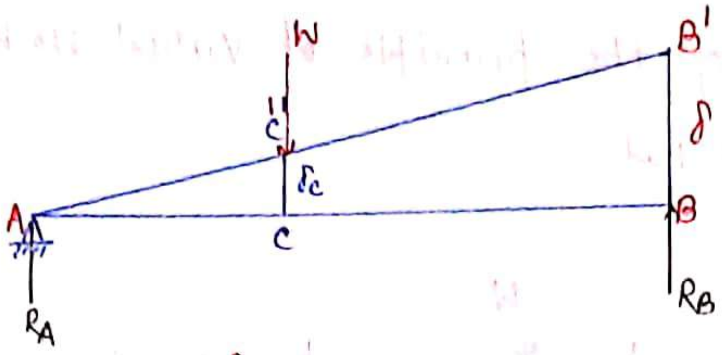

Fig:

Let δ_c be the upward virtual displacement of the beam at C due to the point load.

Now, in two similar triangles ABB' and ACC'

$$\cancel{\frac{\delta c}{\delta\delta}} \qquad \frac{\delta}{l} = \frac{\delta_c}{a} \qquad \therefore \delta_c = \frac{a}{l}\delta \ (\uparrow)$$

It is clear that:
R_B has +ve (positive) virtual work done $= +R_B \cdot \delta$
W has −ve (negative) virtual work done $= -W \cdot \delta_c$
whereas R_A does not have any work done. $= R_A \times 0$

We know that from the principle of virtual work, $\delta U = 0$

$$\therefore \quad -W \times \delta_c + R_B \times \delta = 0$$

or, $R_B \times \delta = W \times \frac{a}{l}\delta$

$$\therefore R_B = W\frac{a}{l} \uparrow$$

Similarly, keeping B in its position, it can be proved that the vertical reaction at A;

$$R_A = W\frac{b}{l}\ (\uparrow)$$

<u>Ans.</u>

Application of the principle of virtual work for beams carrying uniformly distributed load :

Consider a beam AB of length l simply supported at its both ends, and carrying a uniformly distributed load w per unit length for the whole span as shown in fig---

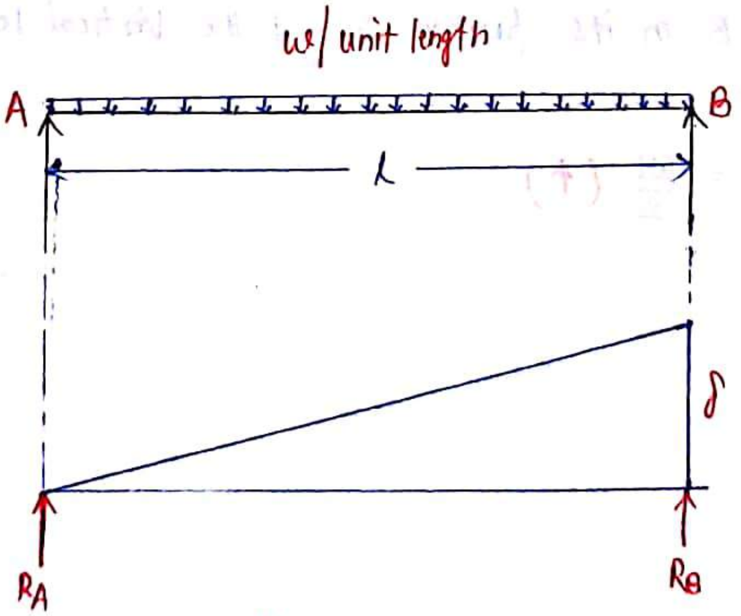

Fig :

Now, Consider an upward virtual displacement δ of the beam at B. Keeping A in its position, draw a displacement diagram as shown in fig - - - - .

from above diagram, it is clear that

Virtual work done by R_A = $R_A \times 0$

Virtual work done by R_B = $+R_B \times \delta$ (↑)

Virtual work done by w = $-w\left(\frac{0+\delta}{2} \times l\right)$ (↓)

We know that from the principle of virtual work-done

i.e $\delta U = 0$

$\therefore \quad R_B \times \delta - \frac{1}{2} w \cdot \delta \times l = 0$

or, $R_B = \frac{wl}{2}$ (↑)

Similarly, Keeping B in its position, we get the vertical reaction at A;

$R_A = \frac{wl}{2}$ (↑)

- - -

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Prob: A simply supported beam AB of span 5 m is loaded as shown in fig.... . Using the principle of virtual work, find the reactions at A and B.

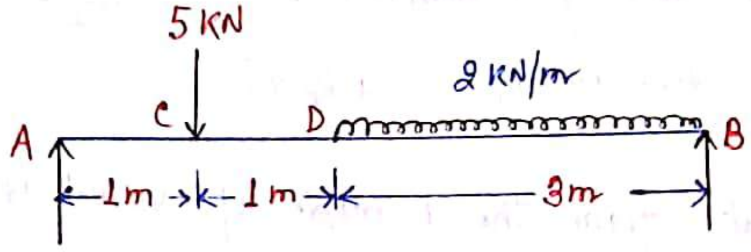

fig:

Solu^n:

let

R_A = reaction at A

R_B = reaction at B; and

δ = virtual upward displacement of the beam at B.

Keeping A in its position, draw displacement diagram

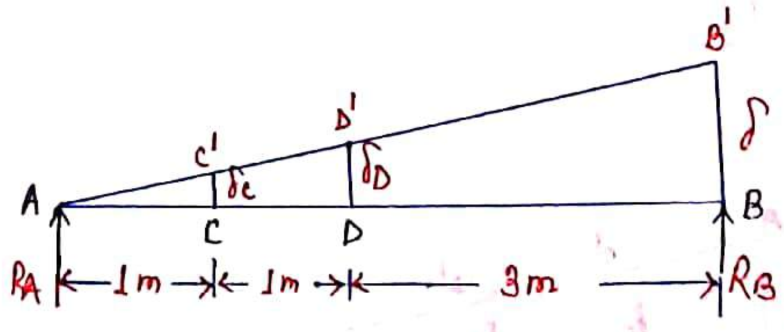

fig:

from the geometry of the figure; $\delta_D = \frac{2}{5}\delta$

$\delta_C = \frac{1}{5}\delta$

It is clear that;

Virtual work done by $R_A = R_A \times 0$

Virtual work done by $5\,KN = -5 \times \delta_C$

Virtual work done by 2 KN/m (u.d.l) $= -2 \times \left(\frac{\delta_D + \delta}{2} \times 3\right)$

Virtual work done by $R_B = + R_B \times \delta$

We know that from the principle of virtual work done

i.e. $\delta U = 0$

$$\therefore -5 \times \delta_C - 2 \times \left(\frac{\delta_D + \delta}{2} \times 3\right) + R_B \times \delta = 0$$

$$\text{or, } -5 \times \frac{\delta}{5} - 2 \times \left(\frac{\frac{2}{5}\delta + \delta}{2} \times 3\right) + R_B \times \delta = 0$$

or, $R_B = 5.2\ KN$ ---- Ans.

Similarly, keeping B in its position; draw a displacement diagram as shown in fig:

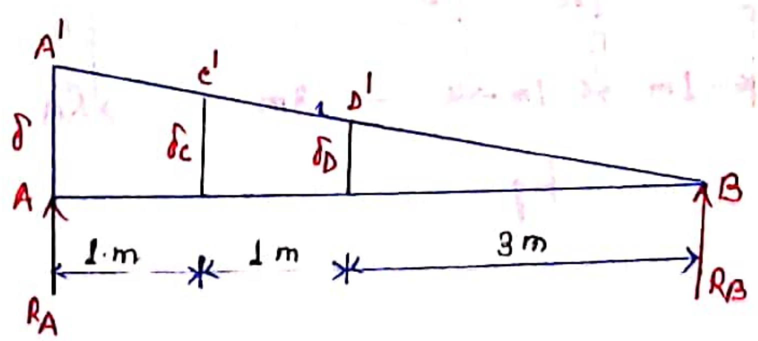

Fig:

let δ : virtual upward displacement of beam at A.

from the geometry of figure,

$$\delta_C = \frac{4}{5}\delta$$

$$\delta_D = \frac{3}{5}\delta$$

It is clear that:

virtual work done by R_A = $+R_A \cdot \delta$

virtual work done by 5 KN = $-5 \times \delta_C$

virtual work done by 2 KN/m = $-2 \times \left(\frac{\delta_D + 0}{2} \times 3\right)$

virtual work done by R_B = $R_B \times 0$

∴ from the principle of virtual work done;

i.e. $\delta U = 0$

$$\therefore \quad R_A \times \delta - 5\,\delta_C - 2 \times \frac{3}{2}\delta_D = 0$$

or, $R_A \times \delta - 5 \times \frac{4}{5}\delta - 3 \times \frac{3}{5}\delta = 0$

$$\therefore R_A = 5.8 \text{ KN}$$ .... Ans.

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(Prob) Using principle of virtual work, find the reaction R_D for the system shown in fig. for the vertical load of 200 N acting on the compound beam.

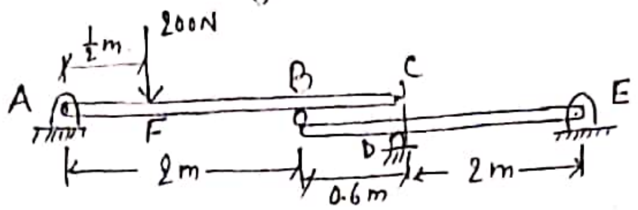


Fig:

Solu^n:

Let reactions at A, D, and E be R_A, R_D and R_E respectively.

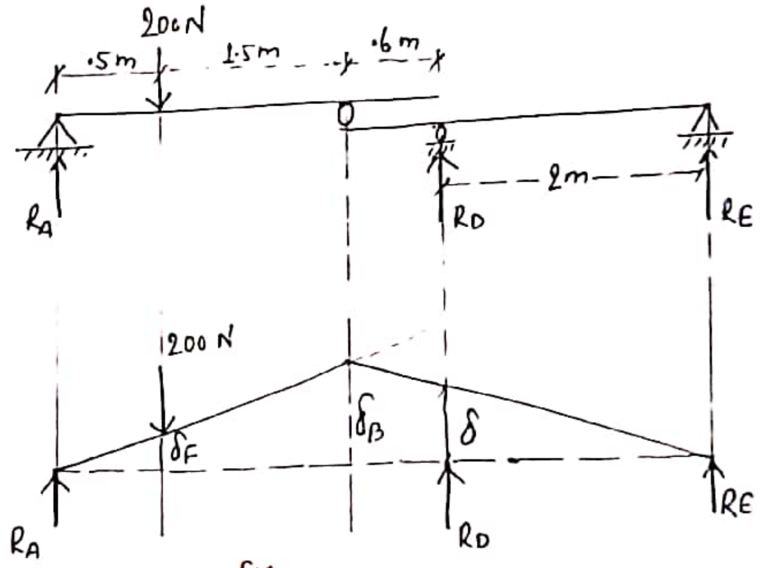


Fig:

Now, give virtual displacement δ at point D upward, and maintain geometrical consistency of beam,

from similar triangles; $\delta/2 = \dfrac{\delta_B}{2.6} \Rightarrow \delta_B = 1.3\delta$

and; $\dfrac{\delta_F}{0.5} = \dfrac{\delta_B}{2}$

$\therefore \delta_F = \dfrac{\delta_B}{4} = \dfrac{1.3\delta}{4}$

$\delta_F = 0.325\,\delta$

Using principle of Virtual Work;

for the System; net Virtual Work = 0

$\therefore \quad 200 \times (-\delta_F) + R_D(\delta) = 0$

$\therefore R_D\,\delta = 200 \times (0.325)\delta$

$\therefore \boxed{R_D = 65\ N\,(\uparrow)}$ This is required reaction.

(Prob) Determine the reaction at B and D for the beam shown:
Using principle of virtual work.

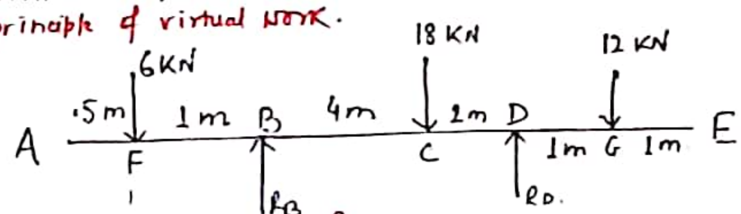
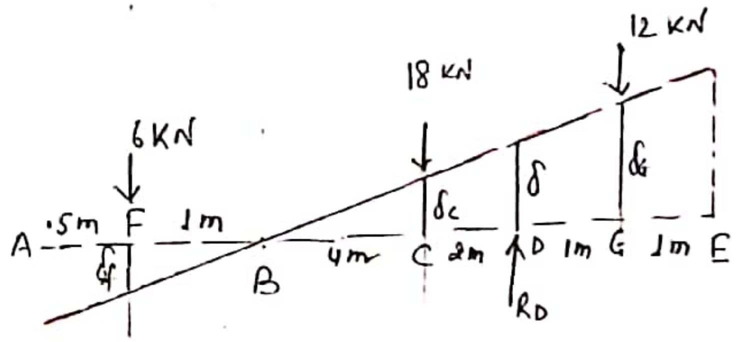


fig:

Ans: Consider a small virtual displacement (δ) at D; keeping B in its position.



Using principle of virtual work;

for the System; net Virtual Work = 0

$(+6)\times(\delta_F) - 18\times\delta_C + R_D\times\delta - 12\times\delta_G = 0$

Where; $\frac{\delta}{6} = \frac{\delta_F}{1} \Rightarrow \delta_F = \delta/6$

$\delta/6 = \frac{\delta_C}{4} \Rightarrow \delta_C = 2/3\,\delta$

$\delta/6 = \frac{\delta_G}{7} \Rightarrow \delta_G = 7/6\,\delta$

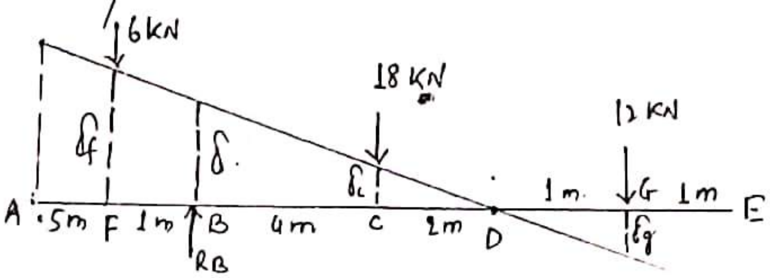
$$+6\times(+\delta/6) - 18\times \tfrac{2}{3}\delta + R_D\times\delta - 12\times \tfrac{7}{6}\delta = 0$$

$$\therefore +1 + 12 - 14 + R_D = 0$$

$$\therefore R_D - 25 = 0$$

$$\boxed{R_D = 25\ KN}$$ Ans

Consider a small virtual displacement δ at B; keeping D in its position.



Applying the principle of virtual work;

$$(-6)\times\delta_f + R_B\times\delta - 18\times\delta_c + (12)\times\delta_g = 0$$

where $\frac{\delta_f}{7} = \frac{\delta}{6} \Rightarrow \delta_f = \tfrac{7}{6}\delta$

$\frac{\delta_c}{2} = \frac{\delta}{6} \Rightarrow \delta_c = \tfrac{1}{3}\delta$

$\frac{\delta_g}{1} = \frac{\delta}{6} \Rightarrow \delta_g = \tfrac{1}{6}\delta$

$$\therefore (-6)\times \tfrac{7}{6}\delta + R_B\times\delta - 18\times\tfrac{1}{3}\delta + 12\times(\tfrac{1}{6}\delta) = 0$$

$$-7 + R_B - 6 + 2 = 0$$

$$R_B = -2 + 13 = 11$$

$$\boxed{R_B = 11\ KN}$$ Ans